Power Efficient Throughput Maximization in Multi-hop Wireless Networks

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***Abstract*— We study the problem of total throughput maximization in arbitrary multi-hop wireless networks, with constraints on the total power usage (denoted by PETM), when nodes have the capability to adaptively choose their power levels, which is the case with software defined radio devices. The underlying interference graph changes when power levels change, making PETM a complex cross-layer optimization problem. We develop a linear programming formulation for this problem, that leads to a constant factor approximation to the total throughput rate, for any given bound on the total power usage. Our result is a rigorously provable worst case approximation guarantee, which holds for any instance. Our formulation is generic and can accommodate different interference models and objective functions. We complement our theoretical analysis with simulations and compute the explicit tradeoffs between fairness, total throughput and power usage.**

I. Introduction

Power usage is usually a severe constraint in multi-hop wireless networks, because of limited infrastructure support. Opportunistic or adaptive power control can be very helpful in improving the energy utilization, and recent advances in software defined radio (SDR) technology allow nodes to switch transmission power levels with low overhead [10]. In this paper, we study the problem of joint optimization of the total throughput rate and the total power used in an arbitrary multi-hop wireless network, in which nodes have the capability to adaptively choose power levels. Formally, given a set of nodes V, a set of connections $\mathcal{C}$, a set $\mathcal{J}$ of possible power levels to use, and a bound B on the total power, the objective of the *Power Efficient Throughput Maximization* (PETM) problem is to maximize the total throughput rate and ensure that the total power used is at most B, by choosing: (a) **routes** for each connection, (b) an interference free **schedule** for all the links, (c) the **power level** for each link at each time, and (d) the **rates** for each connection. Observe that this is a cross-layer optimization problem, involving constraints from the Physical, MAC and Routing layers. The scheduling and power control steps are coupled in a complex manner because of interference - the set of edges that interfere with any edge e, depends, *not only on the power level used on* e*, but also on the power levels used on other close-by edges* (see Figure 1), and the link capacities depend on the transmission power levels.

Most of the prior work on provable algorithms for power control has ignored its adaptive aspect. One of the papers that is most closely related to this work is by Bhatia and Kodialam [3], who study the PETM problem with the assumption that the interference graph (the graph that specifies the set of edges interfering with any edge) is fixed initially, i.e., varying the power levels only changes the link capacities, but not the interference graph. However, even this problem remains very non-trivial, as shown by Bhatia and Kodialam [3]. An important consequence of these assumptions is that the power level for any link in the optimum solution does not vary with time and, therefore, it suffices to focus on non-adaptive power control. This is no longer true for the PETM problem in the general setting - we show that there are instances in which the transmission power level on edges have to be changed adaptively in order to achieve the optimal throughput rate, and there is a significant reduction in the capacity by requiring fixed power levels for edges. On the flip side, adaptive power control increases the search space of the PETM problem, thereby making it a complex and challenging problem to solve in polynomial time.

Our Contributions: In this paper, we study provable algorithms for the PETM problem for arbitrary wireless networks. Our main results are:

1. We develop a new linear programming (LP) formulation to approximate the PETM problem, which incorporates constraints from physical, MAC and routing layers. We show that this program can be used to derive both necessary and sufficient conditions for the optimum rate vector for the PETM problem for any arbitrary wireless network. A key idea underlying our formulation is to consider tuples involving an edge and the transmission power level on it, instead of just the edges. Our formulation guarantees that the total throughput achieved is at least r_{opt}/μ, where r_{opt} is the optimum throughput rate and μ is a constant and depends on the interference model used. This LP formulation can have exponential size, and our algorithm involves another relaxation, which can be solved in polynomial time, and guarantees a throughput of at least $r_{opt}/(1+\epsilon)\mu$, where $\epsilon > 0$ is any constant - the running time increases as a polynomial function of $1/\epsilon$. Moreover our formulation is generic and can accommodate different interference models and objective functions, such as long term fairness, and gives quantitative trade-offs between these objectives. This formulation is much simpler than the non-convex program formulated in [3], and is the first such result for the PETM problem. Also, in contrast to the result of [3], we show that there are instances in which the optimum solution that maximizes the total throughput capacity requires adaptive power control (see Figure 1).

2. We evaluate the performance of our algorithms by solving the linear programs using the NEOS solvers [1], and compute explicit tradeoffs between various quantities such as the total throughput, total power used and the number of connections. Note that it is very difficult to study such tradeoffs directly. We observe empirically that adaptive power control leads to a much higher total throughput as compared to the non-adaptive setting. We observe that by enforcing fairness among various connections, the throughput varies significantly. The total throughput reaches a saturation point as the total power bound is increased. This allows us to understand the capacity limitations of the system.

The primary contribution of this paper is theoretical, and our result gives the first provable algorithm for the PETM problem in a general cross-layer setting. One use of our algorithm would be to get good upper and lower bounds on the optimum system performance, and use that to compare and quantify the performance of protocols in practice. The techniques developed in this paper build on the work by [8] in a non-trivial manner - the results of [8] involve fixed power levels, and as mentioned earlier, our formulations involve edge-power transmission tuples (defined in Section III). Because of the limited space, we have omitted some of the proof details.

II. Related Work

Cross-layer design has been an extremely active area of research in the recent years. Recent work by [4] presents an integrated routing, link scheduling and power allocation policy for minimizing total average power in a multi-hop wireless network. Their algorithm uses interesting techniques and appears to be efficient in practice. However as acknowledged in [4], it can have a worst case exponential complexity in the number of transmission nodes. In a similar vein of research, [5] considers the problem of joint scheduling and power control for multi-hop networks for several different interference models. They provide centralized as well as distributed algorithms for scheduling and power control. The authors of [11] further extend the work by [5] for the case of multicasting. Most of the algorithms provided in [5], [11] are greedy and heuristic. Also as demonstrated in [9], these algorithms can lead to a sub-optimal solution. Our work is most closely related to [3] and [8]. In [3], the authors study the joint routing, scheduling and power control problem with the objective of minimizing total power consumed. They formulate the overall problem as a non-convex optimization problem with non-linear constraints and develop a polynomial time 3-approximation algorithm for solving this problem. There are crucial differences between the work presented here and the work presented in [3]- (i) the interference graph used in [3] is assumed to be fixed, whereas we allow this to change with the power levels, (ii) we consider general interference models that take into account both primary and secondary interference, whereas, [3] only considers primary interference, (iii) our algorithm uses a linear program, which can be solved much more efficiently than the quadratic program discussed in [3]. The result of [8] study the joint optimization problem of routing and scheduling with the objective of maximizing the throughput - they do not consider the issue of power control, and it is not intuitive to directly extend their framework to solve the PETM problem.

PSfrag replacements

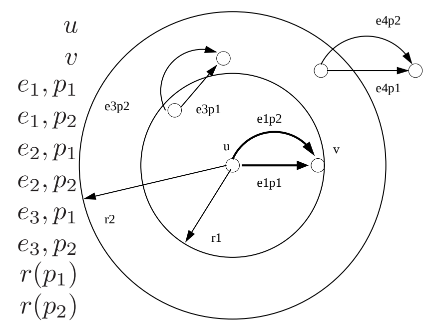


Fig. 1. Illustrating the change in the interference graph with change in transmission power levels (see Section III for the definitions): each node has two possible power levels p_1 and p_2, and let c_1 and c_2 denote the link rates corresponding to these power levels. The rings of radii $r(p_1), r(p_2)$ represent transmission ranges for node u for power levels p_1, p_2. Using the Tx-model, we have, $I(e_1, p_1) = \{(e_1, p_2), (e_2, p_1), (e_2, p_2)\}$ and $I(e_1, p_2) = \{(e_1, p_1), (e_2, p_1), (e_2, p_2), (e_3, p_1), (e_3, p_2)\}$, and so on. Suppose we want to maximize the flow on e_1, and require a rate of $c_1/3$ on link e_3. Then, the optimal solution is to have link rates $x(e_1, p_1) = x(e_3, p_1) = 1/3$ and $x(e_1, p_2) = 1/3$, which leads to a total rate of $c_1/3 + 2c_2/3$ on e_1, instead of c_1 or $2c_2/3$, which is possible by fixing the power level on e_1 to be fixed at p_1 or p_2, respectively.

III. Preliminaries and Model

Network Model: We describe an input instance as PETM$(V, \mathcal{C}, \mathcal{J}, B)$, where: (i) V is a set of n nodes, located on the Euclidean plane, (ii) $\mathcal{C}$ denotes a set of connections, with the ith connection from node s_i to node t_i, (iii) $\mathcal{J}$ denotes the set of possible power levels, with J_{min} and J_{max} being the minimum and maximum possible power levels, respectively, and $\Gamma = J_{max}/J_{min}$, and (iv) B denotes the bound on the total power usage. For a node $u \in V$ transmitting at power level p, we assume that the signal propagation is omni-directional. Another standard assumption we make is that the signal decays as $r^{-\alpha}$ with distance r, where $\alpha > 2$ [6] is the path loss exponent. We use the disk graph model, in which the signal from node u transmitting at power p is received only within a transmission range of $range(p) = (\frac{p}{c_1 N_0})^{1/\alpha}$, where c_1 is a constant and N_0 denotes the noise spectrum density. This implies that a transmission is possible on edge $e = (u, v)$ only if node u uses a transmission power p such that $d(u, v) \leq range(p)$, where $d(u, v)$ denotes the Euclidean distance between node u and node v (cf. Fig. 1). We define set $E = (V \times V)$ as a set of all possible links, i.e., $e = (u, v) \in E$ if $d(u, v) \leq range(J_{max})$. We use the additive white Gaussian noise (AWGN) channel model for specifying the link capacities. In this model, the capacity of a link $e = (u, v)$, where u transmits at power level p is given as [3] $cap(e,p) = W \log_2\left(1 + \frac{p\sigma(e)}{N_0 W}\right)$, where $\sigma(e) = 1/d(u,v)^\alpha$, and W is the bandwidth. In reality, this is an upper bound on the link capacity, and $cap(e,p)$ depends on the SINR, which is determined by all the other transmissions. However, as pointed out in [3], the PETM problem remains non-trivial even under this simplifying assumption. Since the link capacities depend on the transmission power levels, we consider transmission tuples (e,p) consisting of the link and power level (cf. Fig. 1). Let $\mathcal{T} = \mathcal{T}(\mathcal{J}, E) = \{(e = (u,v), p) \in E \times \mathcal{J} : d(u,v) \leq range(p)\}$ denote the set of all possible feasible transmission tuples. We define $N_{out}(u) = \{(e = (u,v'), p) : e \in V \times V, p \in \mathcal{J}\}$ and $N_{in}(u) = \{(e = (v',u), p) : eV \times V, p \in \mathcal{J}\}$ as a set consisting of outgoing and incoming transmissions to node $u \in V$ respectively.

Interference Model: We extend the usual notion of interference among links to that involving transmission tuples. For edges $e = (u,v), e' = (u',v')$, the tuples (e,p) and (e',p') interfere with each other if either (i) node u' or v' is within *interference range* of u, or (ii) node u or node v is within interference range of node u'. In the Tx-model, the interference range associated with (e,p) is $(1+\Delta)range(p)$ [8], where Δ is a constant parameter. For $e = (u,v)$, we define $I(e,p) = I_{pri}(e,p) \cup I_{sec}(e,p)$ to be the set of transmission tuples that interfere with (e,p); this set is partitioned into primary and secondary interference sets $I_{pri}(e,p)$ and $I_{sec}(e,p)$. Primary interference set for (e,p) consists of transmissions that share a common end-point with the an edge e. $I_{pri}(e,p) = \{(e' = (u',v'), p') : e' \in V \times V,\ p' \in \mathcal{J} \wedge e' \in N(u) \cup N(v)\}$. We consider the secondary interference set for any (e,p) according to the Tx-model. $I_{sec}(e,p) = \{(e' = (u',v'), p') : e' \in V \times V,\ p' \in \mathcal{J} \wedge (e',p') \notin I_{pri}(e,p) \wedge d(u,u') \leq (1+\Delta)(range(p) + range(p'))\}$. Let $I_{sec\geq}(e,p) = \{(e' = (u',v'), p') \in I_{sec}(e,p) : p' \geq p/2\}$, and $I_{\geq}(e,p) = I_{pri}(e,p) \cup I_{sec\geq}(e,p)$ (cf. Fig. 1). Note that the interference relations are symmetric, therefore if $(e',p') \in I(e,p)$ then $(e,p) \in I(e',p')$. Similar secondary interference sets can be defined for the Protocol model [8].

Link rates and feasible schedules. We assume that time is divided into slots of uniform length τ, the system operates in a synchronous mode and the transmissions occur on an error-free channel. Let r_i denote the rate on connection i, and $\mathbf{r}$ denote the total rate (throughput) achieved over all the connections. We define flow and utilization vectors over transmission tuples instead of links. Let $f_j(e,p)$ denote the mean transmission rate on the transmission tuple (e,p) for connection j, and let $f(e,p) = \sum_j f_j(e,p)$ - this is the mean rate at which link e transmits using power level p. Let $x(e,p) = f(e,p)/cap(e,p)$ denote the mean utilization fraction for this transmission tuple, and let $\vec{x}$ denote the utilization vector. The (long term) fairness index $\lambda \in [0,1]$ is defined to be the ratio between the minimum and maximum rates, i.e. $\lambda = \dfrac{\min_i r_i}{\max_i r_i}$. An end-to-end schedule $\mathcal{S}$ describes the specific times at which transmissions are done. Let $X(e,p,t)$ be an indicator variable which is 1 if the transmission (e,p) is scheduled by $\mathcal{S}$ at time t. Then, the utilization fraction $x(e,p)$ corresponding to this schedule is $x(e,p) = \lim_{T\to\infty} \sum_{t\leq T} X(e,p,t)/T$. The schedule $\mathcal{S}$ is interference-free at any time t, if for any $(e,p) \in \mathcal{T}$, $X(e,p,t) = 1$ and $X(e',p',t) = 0$ for all $(e',p') \in I(e,p)$. We say that a utilization vector $\vec{x}$ is feasible if there is an interference-free schedule $\mathcal{S}$ that achieves it. The objective of the PETM problem is to find a feasible utilization vector $\vec{x}$ so that the total throughput rate $\sum_i r_i$ corresponding to it is maximized, and the total power consumed is within a certain bound B.

IV. An approximation algorithm for the PETM problem

We develop a linear relaxation for the PETM problem here. A key idea of our approach is to use linear constraints involving transmission tuples to develop necessary and sufficient conditions for the feasible rate region.

A. Link Rate Stability: Necessary Conditions

The following lemma describes constraints that must be satisfied by any feasible utilization vector $\vec{x}$. It crucially uses the inductive ordering on transmission tuples.

Lemma 1: Any feasible link utilization vector $\vec{x}$ satisfies the following constraints, $\forall (e,p) \in \mathcal{T}$,

$$\sum_{(e',p')\in I_{pri}(e,p)} x(e',p') + \sum_{(e',p')\in I_{sec\geq}(e,p)} x(e',p') \leq \mu.$$

Proof: (Sketch) Consider a valid schedule $\mathcal{S}$. As defined earlier, let $X(e,p,t)$ be the indicator variable for this schedule, which is 1 if the transmission tuple (e,p) is scheduled in $\mathcal{S}$ at time t. A key property of any feasible schedule is:

$$\forall (e,p) \in \mathcal{T}, \forall t, \sum_{(e',p')\in I_{sec\geq}(e,p)} X(e',p',t) \leq \mu,$$

where μ is a fixed constant that depends only on the interference model; for the Tx-model, $\mu = 5$. We skip the proof of this property, which is based on the techniques developed in [8].

Next, during any time slot, exactly one of the following events occur: (1) Transmission (e,p) is active: In this case none of the other transmissions (excluding transmission (e,p)) from the set $I_{pri}(e,p) \cup I_{sec\geq}(e,p)$ can be active.
(2) Transmission (e,p) is inactive: From the above property, at most μ transmission tuples from the set $I_{sec\geq}(e,p)$ can be active at time t. Combining all of the above, we get, $\forall (e,p) \in \mathcal{T}, \forall t$,

$$\sum_{(e',p')\in I_{pri}(e,p)} X(e',p',t) + \sum_{(e',p')\in I_{sec\geq}(e,p)} X(e',p',t) \leq \mu.$$

Therefore, for any time T, we have, $\forall (e,p) \in \mathcal{T}$,

$$\sum_{t\leq T} \Big(\sum_{(e',p')\in I_{pri}(e,p)} X(e',p',t) + \sum_{(e',p')\in I_{sec\geq}(e,p)} X(e',p',t) \Big) \leq \mu T.$$

Dividing both sides by T, and using the fact that $\lim_{T\to\infty} \sum_{t\leq T} X(e,p,t)/T = x(e,p)$, the Lemma follows. ∎

Note that the constraint in the above lemma only involves the transmission tuples in $I_{sec\geq}(e,p)$, instead of $I(e,p)$ - this is crucial, because it is easy to construct instances in which $\sum_{(e',p')\in I_{pri}(e,p)} x(e',p') + \sum_{(e',p')\in I_{sec}(e,p)} x(e',p')$ is unbounded.

B. Link-Rate Stability: Sufficient Conditions

Utilization vectors $\vec{x}$ satisfying the constraints of Lemma 1 are not necessarily feasible, i.e., there may not exist a feasible schedule that could lead to the utilization vector $\bar{x}$. We develop conditions under which $\bar{x}$ can be scheduled feasibly. We describe a centralized greedy algorithm (cf. Algorithm 1) for constructing a feasible schedule.

We assume that time is divided into uniform and contiguous windows or frames of length w and w.l.o.g. $w \cdot x(e,p)$ is integral for all $(e,p) \in \mathcal{T}$. Algorithm Schedule constructs a periodic schedule $\mathcal{S}$, by repeating a schedule $\mathcal{S}_M$ for every frame M. Within each frame M, the algorithm assigns a set of $s(e,p)$ slots for every transmission (e,p), where $|s(e,p)| = w \cdot x(e,p)$, in such a way that no two interfering transmissions are assigned the same time slot. For example, if $(e',p') \in I_{\geq}(e'',p'')$, then $s(e',p') \cap s(e'',p'') = \phi$. The schedule $\mathcal{S}_M$ is constructed by combining schedules from all the frames. For the algorithm to be stable, we need to find conditions under which, step 7 of the algorithm would be successful. The following lemma gives sufficient conditions for feasible utilization vectors and shows that the the algorithm is indeed successful.

Lemma 2: Any utilization vector $\vec{x}$ satisfying the following conditions can be scheduled feasibly, $\forall (e,p) \in \mathcal{T}$,

$$\sum_{(e',p')\in I_{pri}(e,p)} x(e',p') + \sum_{(e',p')\in I_{sec\geq}(e,p)} x(e',p') \leq 1.$$

Proof: (Sketch) Suppose step 7 of Algorithm Schedule fails for some transmission (e,p), then it must be the case that

$$\sum_{(e',p')\in I_{pri}(e,p)} x(e',p') \cdot w + \sum_{(e',p')\in I_{sec\geq}(e,p)} x(e',p') \cdot w > w.$$

Dividing both sides by w, we get $\sum_{(e',p')\in I_{pri}(e,p)} x(e',p') + \sum_{(e',p')\in I_{sec\geq}(e,p)} x(e',p') > 1$, which contradicts the condition on $\bar{x}$ and the Lemma follows. ■

Algorithm 1: Schedule

Input : (i) $\vec{x}$, (ii) w, (iii) M
Output : Schedule for a frame
1 **for** $(e,p) \in \mathcal{T}$ **do**
2 | $s(e,p) = \phi$
3 **end**
4 Sort transmissions in $\mathcal{T}$ in decreasing order of power levels.
5 **for** *each* $(e,p) \in \mathcal{T}$ *in this order* **do**
6 | $s'(e,p) = \bigcup_{(e',p')\in I_{pri}(e,p)} s(e',p') \bigcup_{(e'',p'')\in I_{sec\geq}(e,p)} s(e'',p'')$
7 | For $s(e,p)$ choose any subset of $M \setminus s'(e,p)$ of size $x(e,p)w$
8 **end**

C. Linear Programming Formulation for PETM

We now put together the constraints from the above sections to obtain a linear program $\mathcal{P}$ for an instance PETM$(V, \mathcal{C}, \mathcal{J}, B)$. Equation 1, defines the link utilization for each transmission. Equation 2 capture the wireless interference. These constraints along with the end-to-end scheduling algorithm discussed previously ensure that flows computed by the LP are feasible. Equations 3 through 6 capture flow conservation constraints and responsible for selecting appropriate routes. Equation 7 ensures that the total power consumed is within a bound B. Equation 8 captures the fairness constraints that ensure that all the flows get a fair share of the throughput. The linear program $\mathcal{P}$ can be efficiently solved using standard solvers such as [1] to obtain the value of r and $x(e,p) \forall (e,p) \in \mathcal{T}$.

$$\max \ r = \sum_{j\in\mathcal{C}} r_j \quad \text{s.t.:}$$

$$\forall (e,p) \in \mathcal{T}, x(e,p) = \frac{f(e,p)}{cap(e,p)} \quad (1)$$

$$\forall (e,p) \in \mathcal{T}, \sum_{(e',p')\in I_{pri}(e,p)} x(e',p') + \sum_{(e',p')\in I_{sec\geq}(e,p)} x(e',p') \leq 1 \quad (2)$$

$$\forall (e,p) \in \mathcal{T}, f(e,p) = \sum_{j\in\mathcal{C}} f_j(e,p) \quad (3)$$

$$\forall j \in \mathcal{C}, \sum_{(e,p)\in N_{out}(s_j)} f_j(e,p) = r_j \quad (4)$$

$$\forall j \in \mathcal{C}, \forall u \neq s_j, t_j \sum_{(e,p)\in N_{out}(u)} f_j(e,p) = \sum_{(e,p)\in N_{in}(u)} f_j(e,p) \quad (5)$$

$$\forall j \in \mathcal{C}, \sum_{(e,p)\in N_{in}(s_j)} f_j(e,p) = 0 \quad (6)$$

$$\sum_{(e,p)\in\mathcal{T}} x(e,p) \cdot p \leq B \quad (7)$$

$$\forall i \in \mathcal{C}, \forall j \in \mathcal{C}\backslash\{i\}, r_i \geq \lambda \cdot r_j \quad (8)$$

Theorem 1: Let r_{opt} denote the maximum total throughput rate achievable for a given PETM instance. The optimum solution $\vec{x}$ to linear program $\mathcal{P}$ is feasible and leads to a total throughput rate of at least r_{opt}/μ, where μ is a constant, depending on the interference model.

Proof: The fact that $\vec{x}$ can be scheduled feasibly follows from Lemma 2. Let $\vec{x}_{opt}$ denote an optimum rate vector, which achieves a total throughput of r_{opt}. By Lemma 1, $\vec{x}_{opt}$ satisfies all the necessary conditions. Since all the constraints, including the power constraint, are linear, the vector $\frac{1}{\mu}\vec{x}_{opt}$ satisfies the program $\mathcal{P}$. Since $\vec{x}$ is the optimum solution to $\mathcal{P}$, it follows that the throughput achieved by $\vec{x}$ is at least r_{opt}/μ. ■

D. Putting everything together: a polynomial time solution

The number of variables and constraints in $\mathcal{P}$ is polynomial in number of nodes and size of set $\mathcal{J}$, which could be exponential or unbounded, making it intractable to solve directly. We construct the set $\mathcal{J}' = \{j_1 = j_{min}, j_2 = 2j_1, \ldots, j_{k+1} = 2^k j_1\}$, where k is such that $2^{k-1} j_1 < j_{max} \leq 2^k j_1$, and J_{max} and J_{min} are defined in Section III; we assume that $J_{max}/J_{min} = 2^{O(n)}$, which is a fairly practical constraint.

Theorem 2: The optimum solution to PETM$(V, \mathcal{C}, \mathcal{J}', B)$ described above gives a 2μ factor[1] approximation to the PETM$(V, \mathcal{C}, \mathcal{J}, B)$ problem.

[1]the factor of 2 can be improved to $(1+\epsilon)$ for any $\epsilon > 0$ by replacing 2 in the above discretization process by a suitably chosen $\epsilon' > 0$ that is a function of ϵ.

Proof: Let r_{opt} denote the optimum solution to the PETM problem and let $\vec{r}_{opt}$ be the corresponding rate vector. Let $\mathcal{T} = \mathcal{T}(\mathcal{J})$ and $\mathcal{T}' = \mathcal{T}(\mathcal{J}')$. We first transform the rate vector $\vec{r}_{opt}$ to another rate vector $\vec{z}$ in the following manner: (i) for each $(e,p) \in \mathcal{T}'$: define $g(e,p) = \sum_{p' \in [p,2p)} f(e,p')/(2\mu)$, (ii) for each $(e,p) \in \mathcal{T}'$: $z(e,p) = g(e,p)/cap(e,p)$.
We argue that $\bar{z}$ is a feasible solution to $\mathcal{P}(V, \mathcal{C}, \mathcal{J}', B)$, and achieves a rate of at least $\frac{\bar{r}_{opt}}{2\mu}$. This implies that this program has a non-empty solution space, and the theorem follows.

First, observe that since $cap(e,p) = W \log_2(1 + \frac{p}{N_0 W})$ (see section III), we have, $cap(e,p) \geq cap(e,p')/2$, if $p \leq p' \leq 2p$. Therefore, for any $(e,p) \in \mathcal{T}'$, we have,

$$\begin{aligned}\sum_{p' \in [p,2p)} x(e,p')p' \quad &\geq \quad \sum_{p' \in [p,2p)} \frac{pf(e,p')}{cap(e,p')},\\ &\geq \quad \mu z(e,p)p,\\ &\geq \quad z(e,p)p.\end{aligned}$$

Since the expression for total power used by $\vec{z}$ is $\sum_{(e,p) \in \mathcal{T}'} z(e,p)p$, it follows from the above sequence of inequalities that $\sum_{(e,p) \in \mathcal{T}'} z(e,p)p \leq B$. It remains to show that $\vec{z}$ satisfies all the sufficient conditions for PETM($V, \mathcal{C}, \mathcal{J}', B$) . For any fixed $(e,p) \in \mathcal{T}'$, we observe that

$$\mu \geq \sum_{p' \in [p,2p)} x(e,p') \geq \sum_{p' \in [p,2p)} \frac{f(e,p')}{2cap(e,p)} \geq \mu z(e,p). \qquad (9)$$

The first inequality above follows from the necessary condition corresponding to (e,p), since $\vec{x}$ satisfies the necessary conditions - this implies that the variables $z(e,p)$ are valid utilization variables.

Next, let $\hat{E}(e,p) = \{e' : (e',p') \in I_{\geq}^{(\mathcal{J})}(e,p) \text{ for some } p'\}$ Therefore, we have

$$\begin{aligned}\sum_{(e',p') \in I_{\geq}^{(\mathcal{J})}(e,p)} x(e',p') \quad &= \quad \sum_{e' \in \hat{E}(e,p)} \sum_{p' \in \mathcal{J}'} \sum_{p'' \in [p',2p')} x(e',p'')\\ &= \quad \sum_{(e',p') \in I_{\geq}^{(\mathcal{J}')}(e,p)} \mu z(e',p'),\end{aligned}$$

where the first inequality follows from (9). Therefore, we have $\forall (e,p) \in \mathcal{T}'(\mathcal{J}') : \sum_{(e',p') \in I_{\geq}^{(\mathcal{J}')}(e,p)} z(e',p') \leq 1$.

Therefore it can be seen the $\vec{z}$ satisfies all the sufficient conditions for PETM($V, \mathcal{C}, \mathcal{J}', B$) . Further since
$z(e,p) = \sum_{p' \in [p,2p)} f(e,p')/(2\mu)cap(e,p)$, the Theorem follows.

■

We summarize our approximation algorithm in Algorithm 2

V. SIMULATIONS

We study the tradeoffs between various parameters for specific instances, using our algorithm for the PETM problem. Our main observations are: (i) the total throughput rate with adaptive power control is significantly higher than that for non-adaptive power control; (ii) the total throughput increases with the total power bound initially, but reaches a saturation point if fairness constraints are present; (iii) the system is pretty sensitive to fairness constraints, and there is a sharp decline in the total throughput on increasing the fairness index. Also, in the presence of fairness constraints, increase in the number of connections can lead to a reduction in the total throughput beyond some point.

Algorithm 2: ALG-PETM

Input : (i) V, (ii) $\mathcal{C}$, (iii) $\mathcal{J}$, (iv) B
Output : (i) Total throughput r, (ii) $x(e,p), \forall (e,p) \in \mathcal{T}$ (iii) Schedule $\mathcal{S}$

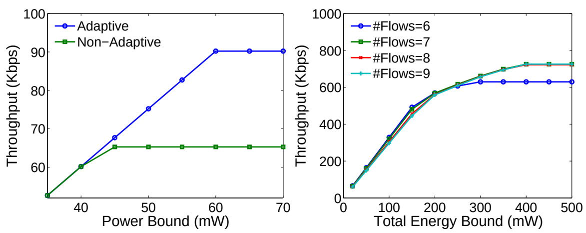
1 Construct set $\mathcal{J}'$ from set J
2 Solve program $\mathcal{P}$ with set $\mathcal{J}'$ using standard LP solvers, to obtain r, and $x(e,p) \forall (e,p) \in \mathcal{T}$
3 Use procedure Schedule to obtain a feasible schedule according to the link utilization

Experimental Setup: All the experiments were conducted on a random distribution of nodes. Except for Experiment #1, we considered a random distribution of 80 nodes on a $2000m \times 2000m$ area. Source destination pairs were chosen uniformly at random. We used $N_0 = 4 \times 10^{-10} mW/Hz, W = 100MHz, c1 = 50000, \alpha = 3$. The set $\mathcal{J}$ consisted of power-levels from $40mW - 140mW$ in increments of $20mW$. We used NEOS [1] solvers for solving all the linear and non-linear programs. We now present the objective and analysis of each experiment.

Experiment #1: Adaptive power control vs. Non-adaptive power control.
The goal of this experiment was to compare the effect of adaptive and non-adaptive power control on the total throughput rate. For the adaptive case, we solve our algorithm from Section IV. We formulate the non-adaptive case as a mixed integer program, with integer variables $X(u,p)$, such that $X(u,p) = 1$ if node u chooses power level p, and solve it exactly. Since this program is difficult to solve exactly in polynomial time, we performed this experiment on a random distribution of 15 nodes on a $500m \times 500m$ area, with 2 source-destination pairs (flows) and 2 choices of power levels (40mW, 60mW) for every node. We varied the total power bound B and observed the corresponding throughput for both the adaptive and non-adaptive case. The fairness index λ was set to 0.
Results and Explanation: Figure 2(a) represents the results of our experiments. It can be seen that adaptive power control is more responsive to the changes in B than non-adaptive power control. Further the throughput achieved by adaptive power control is much higher than that achieved by non-adaptive power control, which reaches its saturation point quickly. In the case of adaptive power levels, we observe that some links choose different power levels at different times. We expect the difference in throughput to be much higher for larger networks with more number of flows.

Experiment #2: Throughput vs. Total power bound .
In this experiment, we varied the total power bound and observed the variation in the throughput for $\lambda = 0.01$. We expected the throughput to increase with B.
Results and Explanation: Figure 2(b) shows the results of our experiments for different values of flows. As expected it can

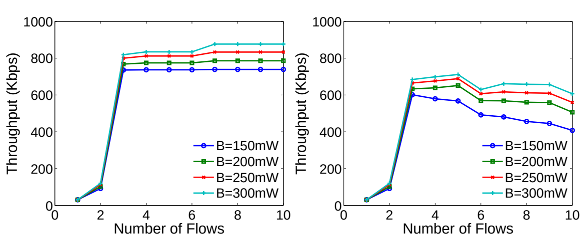
be seen that the throughput increases with the power bound. The throughput however does not increase after a certain value of B and achieves saturation. At this point the system operates at its maximum capacity. This plot is useful to understand the capacity limits of the system and the tradeoff between the throughput achieved and the total power consumed.



(a) $\lambda = 0$ (b) $\lambda = 0.01$

Fig. 2. Throughput(Kbps) vs. total power bound (mW). (a) Experiment #1: Adaptive vs. Non-adaptive power control. (b) Experiment #2: Throughput variation with B for adaptive power control.

Experiment #3:Throughput vs. Number of flows.
In this experiment, we varied the number of flows and observed the variation in the throughput for two different values of fairness index $\lambda = 0$ and $\lambda = 0.01$. Intuitively we expected the throughput to increase with the number of flows.
Results and Explanation: Figures 3(a) and 3(b) plot the results of our experiments for different values of total energy bound B. It can be see that for $\lambda = 0$, after a steep rise, the throughput exhibits a step like behavior and reaches saturation. The variation of throughput with the number of flows is higher for $\lambda = 0.01$. After a steep rise, the throughput starts decreasing with the number of flows. Further it can be seen that the overall throughput is higher when $\lambda = 0$ than $\lambda = 0.01$. Recall that λ is the ratio between the minimum rate and maximum rate across the flows. $\lambda = 0$ implies that there is no fairness in the system and complete starvation of flows could be allowed. $\lambda = 1$ on the other hand implies that all the flows would have identical throughput. When there is no fairness in the system, certain flows are completely starved in order to achieve maximum possible aggregate throughput. The throughput does not vary even if new flows are added to the system. This is because, these new flows are assigned a rate of zero in order to maximize the overall throughput. When fairness constraints are enforced, the overall throughput can decrease in order to ensure that all the flows get their fair share of throughput.



(a) $\lambda = 0$ (b) $\lambda = 0.01$

Fig. 3. Experiment #3: Throughput (Kbps) vs. number of flows with and without fairness constraints.

Experiment #4:Throughput vs. Fairness index.
In this experiment, we varied the fairness index λ from 0 to 1 and observed the variation in the throughput. We fix B to $250mW$ and compute the throughput for different number of flows.

Results and Explanation: Figure 4 shows the results of our experiments for different number of flows. It can be seen that the throughput decreases as λ increases. There is a sharp fall in the throughput as λ varies from 0 to 0.01, and at low λ values, some flows are completely starved.

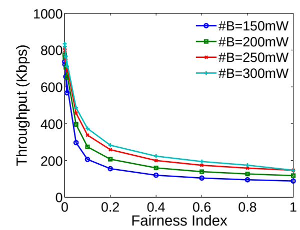


Fig. 4. Experiment #4: Throughput(Kbps) vs. fairness index with #flows=5

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