

TIME-CORRELATION ANALYSIS OF A CLASS OF NONSTATIONARY SIGNALS WITH AN APPLICATION TO RADAR IMAGING

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ABSTRACT

A new method of nonstationary signal analysis, called time-correlation analysis (TCA), is applied to a class of nonstationary random signals containing time distortion. The method reveals a relationship between the TCA summary statistics and the distortion and leads to two nonparametric estimators for the distortion function. An example is given that demonstrates the application of the TCA method to motion compensation problems in radar imaging.

1. INTRODUCTION

Time-correlation analysis (TCA) is a recently proposed technique for nonstationary signal analysis and has been successfully applied to speech processing [1], [2]. The TCA technique is based on the principle of parametric filtering [3] which advocates the combination of judiciously designed filter banks with certain output statistical parameters. With the filter bank, one can enhance the desirable components while suppress the noise and interference; with the output statistics, one can summarize the desirable components with reduced dimensionality and statistical variability. Combining the two can result in very effective signal processing methods, as was demonstrated in [1]-[3].

The gist of the TCA method can be summarized as follows. Suppose that $Y[n]$ is the (discrete-time) random signal to be analyzed and that $\mathcal{H}_\alpha(z) = \sum_{m=0}^{\infty} h_\alpha[m] z^{-m}$ is a filter bank (or a parametric filter) indexed by a parameter α . Let the filtered signal be $Y_\alpha[n] := \mathcal{H}_\alpha(z)Y[n]$. The TCA method proposes to use certain output statistical parameters, such as the autocorrelation coefficient (ACC) $\rho_\alpha[n] := \text{Corr}\{Y_\alpha[n], Y_\alpha[n-1]\}$ or the zero-crossing rate (ZCR)

$\zeta_\alpha[n] := \text{Pr}\{Y_\alpha[n]Y_\alpha[n-1] < 0\}$, as (time-dependent) summary statistics that facilitate the analysis of $Y(t)$. The theory behind the TCA method is as follows. Suppose that the signal is locally stationary, then it may be shown that $\rho_\alpha[n]$ or $\zeta_\alpha[n]$, as a function of α , is able to capture the complete local spectral structure of the signal if the filter bank is judiciously designed. Examples of such filter banks include the first-order all-pole filter $\mathcal{H}_\alpha(z) = 1/(1 - \alpha z^{-1})$.

In this paper, we consider a filter bank which consists of the repeated differences (RD), namely, $\mathcal{H}_k(z) := (1 - z^{-1})^k$, ($k = 0, 1, 2, \dots$). We employ this filter bank to analyze a class of nonstationary signals which take the form

$$Y(t) := X(g(t)), \quad (1)$$

where $X(t)$ is a zero-mean stationary (continuous time) random process and $g(t)$ is a monotone function. This type of nonstationary processes can be found in many applications ranging from radar, sonar, and non-uniform sampling to environmental studies [4]-[6]. For instance, if a radar emits a stationary harmonic signal $X(t)$ to illuminate a point target which moves with an instantaneous velocity $v(t)$ in the direction of the radar, then, due to the Doppler effect, the returned signal $Y(t)$ becomes nonstationary and takes the form of (1) with $g(t) = t - 2(r_0 - \int_0^t v(s) ds)/c$, where c is the velocity of wave propagation and r_0 is the initial distance between the radar and the moving target.

One of the most important problems in these applications is the estimation of the distortion function $g(t)$ based on $Y(t)$. For example, knowledge of $g(t)$ helps obtain optimal (non-uniform) sampling locations for $Y(t)$ and facilitates the modeling of the covariance structure of $Y(t)$. If the distortion results from the Doppler effect of a maneuvering target, as in ISAR imaging (e.g., [7] and [8]), the inverse transform

$$X(t) = Y(g^{-1}(t)) \quad (2)$$

can be used for motion compensation. This method not only reduces the spectral blurring caused by the

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target maneuvering but also maintains the original frequency resolution of the signal.

2. INTERPRETATIONS OF $g(t)$

In the following, we assume that $g(t)$ is strictly increasing with continuous derivative. We also assume that $Y(t)$ is observed in $[0, T]$ and $g([0, T]) = [0, T]$. Under these conditions, it may be shown that the representation (1) is unique for $Y(t)$.

The distortion function $g(t)$ has several interesting interpretations which can be used to derive different estimators. First, let $N(t, \Delta)$ be the number of zero-crossings of $Y(t)$ in the interval $(t - \Delta, t]$, then

$$\zeta(t) := \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} E\{N(t, \Delta)\}$$

represents the expected instantaneous zero-crossing rate of $Y(t)$. It may be shown (e.g., [5] and [10]) that

$$\zeta(t) = c_0 \dot{g}(t),$$

where the constant c_0 denotes the expected ZCR of $X(t)$. Therefore, one can regard $\dot{g}(t)$ as the instantaneous ZCR inflation factor.

Suppose that $X(t)$ is a band-limited process with bandwidth Ω_0 . Then, according to the Shannon sampling theorem, $X(t)$ can be reconstructed (in mean square) from the samples $X(n\Delta)$ with $\Delta := \pi/\Omega_0$ being the critical sampling interval. It may be shown from (1) that the corresponding critical sampling points for $Y(t)$ are determined by $t_n := g^{-1}(n\Delta)$. Since $\Delta_n := t_n - t_{n-1}$ is the local critical sampling interval of $Y(t)$, one may regard π/Δ_n as the local bandwidth of $Y(t)$. It is easy to see that

$$\frac{\pi}{\Delta_n} = \Omega_0 \frac{g(t_n) - g(t_{n-1})}{t_n - t_{n-1}} \doteq \Omega_0 \dot{g}(t_n).$$

Therefore, one can regard

$$\Omega(t) := \Omega_0 \dot{g}(t)$$

as the instantaneous bandwidth of $Y(t)$ and $\dot{g}(t)$ as the instantaneous bandwidth inflation factor.

3. ESTIMATION OF $g(t)$

For any *unknown* constant $a \neq 0$, if $\mu(t) := a \dot{g}(t)$ is estimated from $Y(t)$, then the estimation of $g(t)$ can be easily accomplished by using the transformation

$$g(t) = T \frac{\int_0^t \mu(s) ds}{\int_0^T \mu(s) ds}.$$

In other words, it suffices to estimate the function $\dot{g}(t)$ up to an arbitrary nonzero constant. To this end, we

consider a *nonparametric* approach based on the TCA technique. The nonparametric approach has the advantage of being model-independent and thus has the potential of providing more robust estimators in situations where parametric models of $g(t)$ are inadequate.

Our nonparametric estimators of $g(t)$ are derived from the RD-based TCA technique coupled with the output zero-crossing rates. More precisely, let $Y^{(k)}(t)$, ($k = 0, 1, \dots$), denote the k th derivative of $Y(t)$, which can be approximated by the RD filter output $Y_k[n] := \mathcal{H}_k(z)Y[n]$, where $Y[n] := Y_0[n] := Y(n\Delta)$ are the discrete samples of $Y(t)$. Furthermore, let $N_k(t, \Delta)$ denote the number of zero-crossings of $Y^{(k)}(t)$ in the interval $(t - \Delta, t]$ and let

$$\zeta_k(t) := \lim_{\Delta \rightarrow 0} \frac{1}{\Delta} E\{N_k(t, \Delta)\}$$

be the expected instantaneous ZCR of $Y^{(k)}(t)$. In this case, the TCA method utilizes the $\zeta_k(t)$ as summary statistics for the analysis of $Y(t)$. Under suitable conditions [9], there is a one-to-one correspondence between the ZCR and the ACC of $Y^{(k)}(t)$.

For the estimation of $g(t)$ in particular, we note that

$$\zeta_0(t) = c_0 \dot{g}(t).$$

Therefore, if Δ is sufficiently small so that $Z_0[n] := N_0(n\Delta, \Delta)$ become Bernoulli (0-1) random variables with $Z_0[n] = 1$ if $Y_0[n]Y_0[n-1] < 0$ and $Z_0[n] = 0$ otherwise, then one can write

$$Z_0[n] = \mu_0(n\Delta) + \epsilon_0[n], \quad (3)$$

where

$$\mu_0(t) := E\{N_0(t, \Delta)\} = \Delta c_0 \dot{g}(t) \quad (4)$$

is a deterministic function and $\epsilon_0[n]$ is the (zero-mean) additive noise. Since a constant factor does not affect the estimation of $g(t)$, our task thus becomes the estimation of the mean function $\mu_0(t)$ from the *binary* observations $Z_0[n]$. This can be accomplished with the help of various nonparametric smoothing techniques in statistics.

Our second estimator of $g(t)$ is based on the observation that

$$\dot{Y}(t) = \dot{g}(t) \dot{X}(g(t)).$$

Since $\dot{g}(t) \neq 0$ for all t , the ZCR of $\dot{Y}(t) = Y^{(1)}(t)$ thus coincides with the ZCR of $\dot{X}(g(t))$. By comparing this with the relationship between the ZCR of $X(g(t))$ and $g(t)$, it is not surprising to see that

$$\zeta_1(t) = c_1 \dot{g}(t),$$

where the constant c_1 denotes the expected ZCR of the stationary process $\dot{X}(t)$. Therefore, one can write, for sufficiently small Δ ,

$$Z_1[n] = \mu_1(n\Delta) + \epsilon_1[n], \quad (5)$$

where

$$\mu_1(t) := E\{N_1(t, \Delta)\} = \Delta c_1 \dot{g}(t). \quad (6)$$

is a deterministic function and $\epsilon_1[n]$ is the zero-mean additive noise. Again, as in the first estimation method, smoothing techniques can be employed to obtain nonparametric estimators of the mean function $\mu_1(t)$ from the binary observations $Z_1[n]$.

For the regression problems in (3) and (5), the signal-to-noise ratio (SNR) may be defined as

$$\text{SNR}_i := \frac{\mu_i^2(n\Delta)}{\text{Var}\{\epsilon_i[n]\}} = \frac{\mu_i(n\Delta)}{1 - \mu_i(n\Delta)}$$

for $i = 0, 1$. Clearly, the SNR increases with the increase of $\mu_i(t) \in (0, 1)$. Further, it may be shown by following the lines of [11] that $c_0 \leq c_1$. Therefore the second estimator has the potential of achieving better performance than the first method. This is particularly true if $X(t)$ is dominated by a strong low frequency component. In fact, as a result of the RD highpass filtering, c_1 can be much greater than c_0 so that the “signal” is much more enhanced in $Z_1[n]$.

4. APPLICATION IN RADAR IMAGING

In ISAR (inverse synthetic aperture radar) imaging, it is desirable to construct clear images of a moving target using the radar signals reflected from the target [7], [8]. Conventional ISAR imaging is done by using the Fourier transform of the returned signals. However, due to the possibly complicated maneuvering of the target within the imaging time duration, the returned radar signals may be distorted by the Doppler effect, so that the resulting radar image can become severely blurred if the distortion is not appropriately compensated for.

A simple approach to handling the distortion is to reduce the imaging time duration (e.g., STFT) [8]. A more sophisticated approach calls for the estimation of the distortion followed by suitable transformations to eliminate the blurring. We take the second approach because it preserves the frequency resolution of the original radar signals.

Shown in Fig. 1(a) are the real and imaginary parts of a complex-valued signal representing the time history of a range profile of a maneuvering target (same data as used in [8]). Without correcting the Doppler

distortion which is clearly present in the signal, the Fourier spectrum, shown in the left panel of Fig. 1(e), exhibits blurred peaks. The right panel of Fig. 1(e) shows the Fourier spectrum after the distortion correction (Fig. 1(d)) using a TCA estimator obtained by smoothing the binary zero-crossing process $Z_1[n]$ (Fig. 1(b)). The smoothing is carried out independently for the real and imaginary parts and the results are combined by a simple average to yield the TCA estimators shown in Fig. 1(c). Fig. 1(f) presents the results obtained by smoothing $Z_0[n]$. As we can see, both methods dramatically improve the sharpness of the spectral peaks, but the estimator obtained from $Z_1[n]$ performs slightly better because the RD filtering reduces the dominance of the low frequency component in the signal and thus improves the SNR in the binary process $Z_1[n]$.

REFERENCES

- [1] T. H. Li and J. D. Gibson, “Time-correlation analysis of nonstationary signals with application to speech processing,” *Proc. Int. Symp. Time-Freq. & Time-Scale Anal.*, Paris, France, pp. 449–452, 1996.
- [2] T. H. Li and J. D. Gibson, “Speech analysis and segmentation by parametric filtering,” *IEEE Trans. Audio, Speech Processing*, vol. 4, pp. 203–213, 1996.
- [3] T. H. Li, “Discrimination of time series by parametric filtering,” *J. Amer. Statist. Assoc.*, vol. 91, pp. 284–293, 1996.
- [4] J. K. Hammond and J. Moss, “Time-frequency spectra for nonstationary signals,” in *Nonstationary Stochastic Processes and Their Applications*, A. G. Miamee, Ed. pp. 1–21, Singapore: World Scientific Publishing Co., 1992.
- [5] Y. Y. Zeevi and E. Shlomot, “Nonuniform sampling and anti-aliasing in image representation,” *IEEE Trans. Signal Processing*, vol. 41, pp. 1223–1236, 1993.
- [6] P. Guttorp and P. D. Sampson, “Methods for estimating heterogeneous spatial covariance functions with environmental applications,” in *Handbook of Statistics*, P. Patil and C. R. Rao, Eds., vol. 12, pp. 661–689, Amsterdam: Elsevier Science B. V.
- [7] D. R. Wehner, *High-Resolution Radar, 2nd Edition*, Boston, MA: Artech House, 1995.
- [8] V. C. Chen and S. Qian, “Time-frequency transform vs. Fourier transform for radar imaging,” *Proc. Int. Symp. Time-Freq. & Time-Scale Anal.*, Paris, France, pp. 389–392, 1996.
- [9] B. Kedem, *Time Series Analysis by Higher Order Crossings*, Piscataway, NJ: IEEE Press, 1994.
- [10] H. Cramèr and M. R. Leadbetter, *Stationary and Related Stochastic Processes*, New York: Wiley, 1967.
- [11] B. Kedem and T. H. Li, “Monotone gain, first-order autocorrelation and zero-crossing rate,” *Ann. Statist.*, vol. 19, no. 3, pp. 1672–1674, 1992.

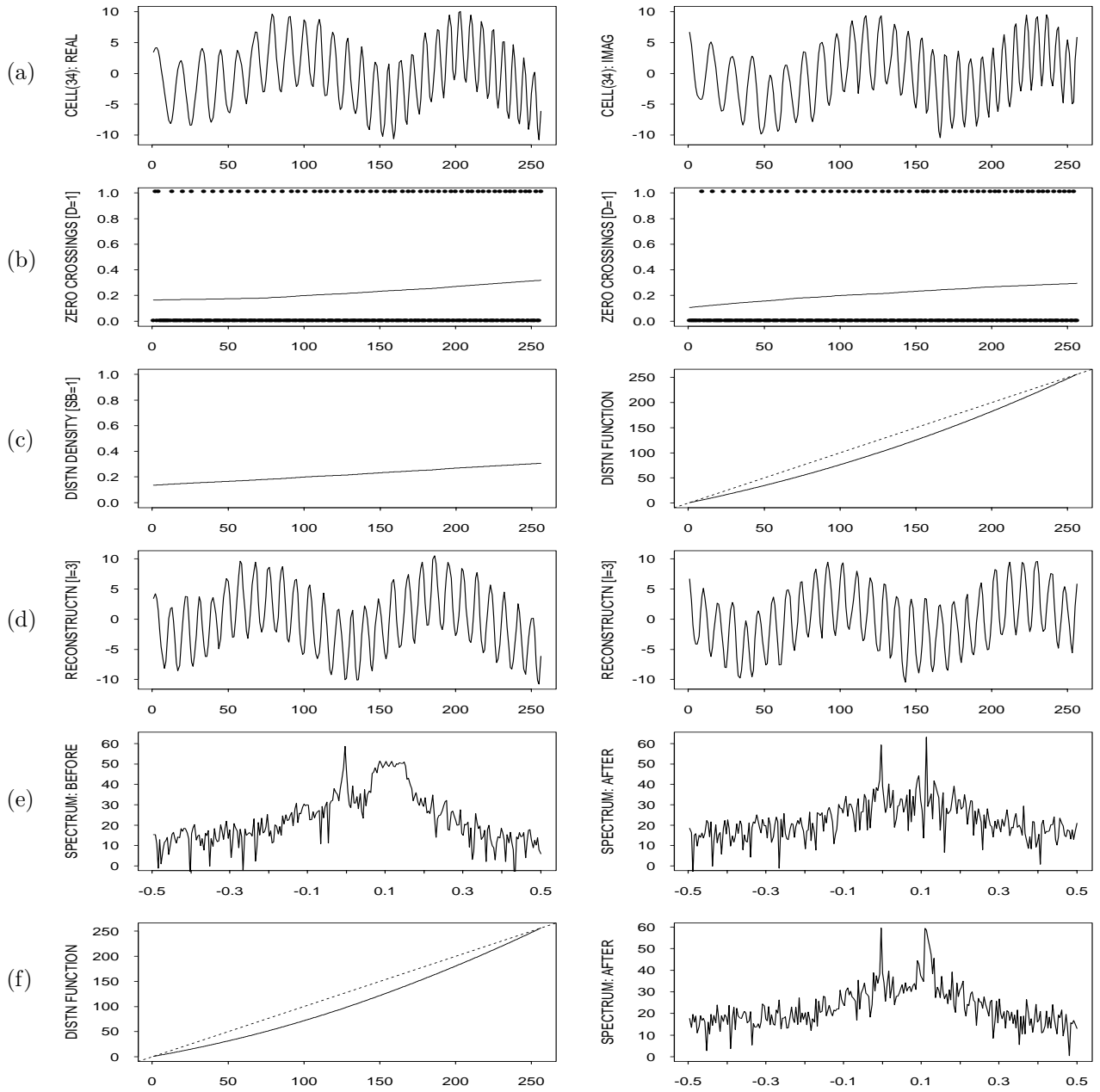


Figure 1: An Example of Correcting Doppler Distortions by the TCA Method. (a) The real and imaginary parts of a distorted signal. (b) The estimated mean function obtained by smoothing the binary process $Z_1[n]$ (represented by the dots) for the real (left) and imaginary (right) parts of the signal. (c) The nonparametric estimate of $\dot{g}(t)$ (left) and of $g(t)$ (right). (d) The real and imaginary parts of the distortion-corrected signal. (e) The Fourier spectrum of the distorted signal (left) and of the distortion-corrected signal (right) – both spectral peaks are sharpened by the distortion-correction procedure. (f) The nonparametric estimate of $g(t)$ (left) obtained by smoothing $Z_0[n]$ and the Fourier spectrum of the resulting distortion-corrected signal (right).