Time-Correlation Analysis of a Class of Nonstationary Signals with an Application to Radar Imaging

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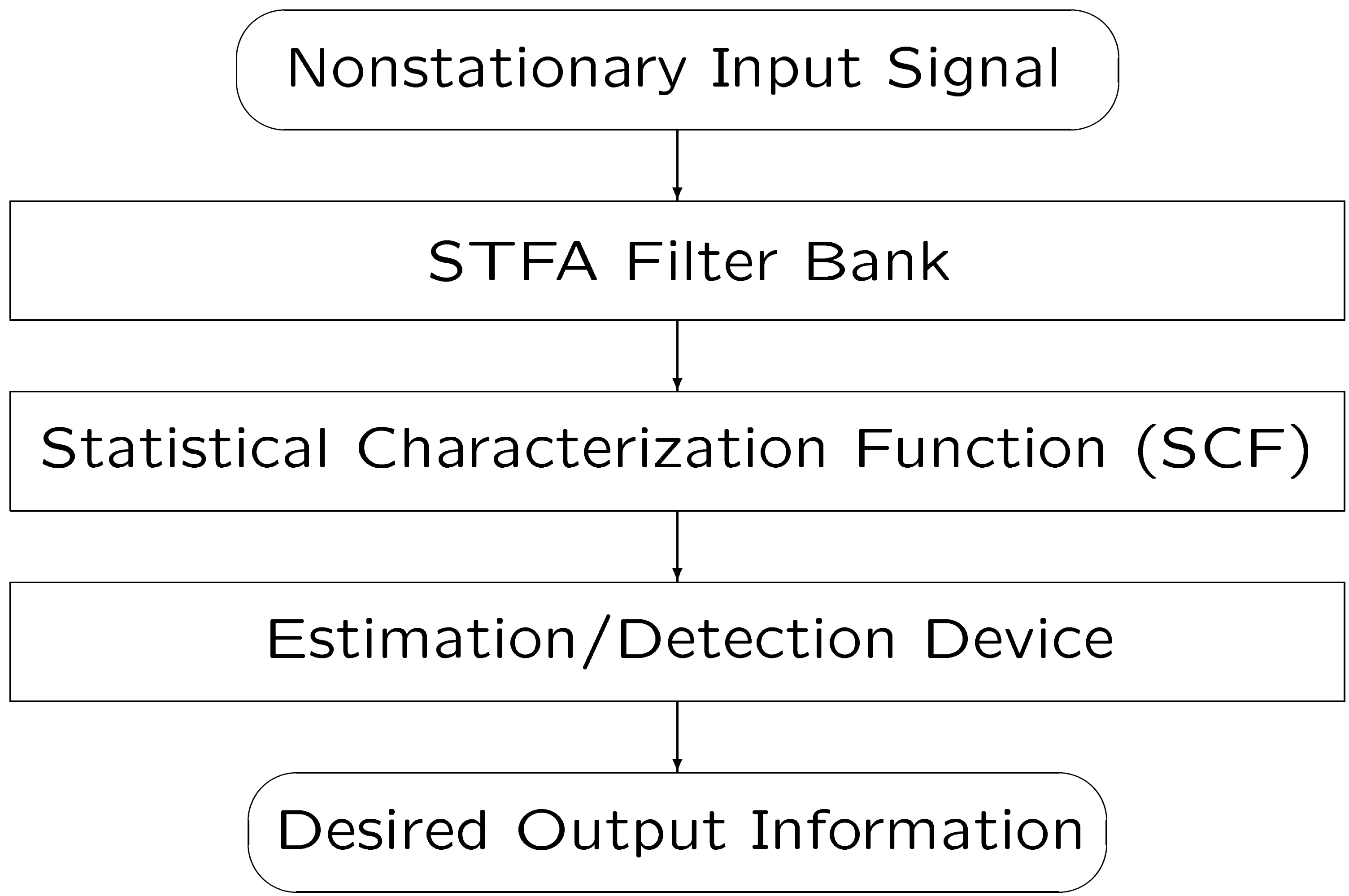
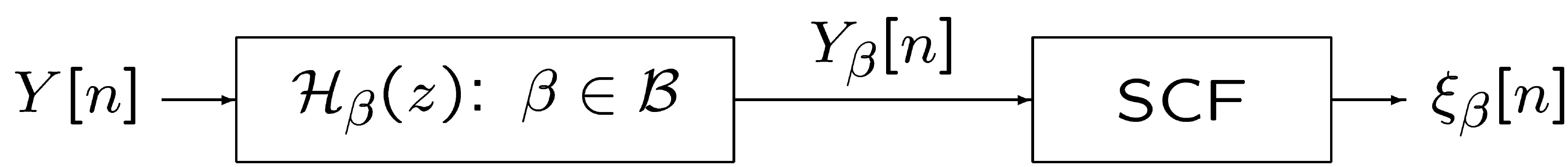
STATISTICAL TIME-FREQUENCY ANALYSIS (STFA)



STFA: FILTER BANK AND SCF



Examples of Filter Banks

- Repeated differencing (RD):

$$\mathcal{H}_k(z) = (1 - z^{-1})^k$$

where $k = 0, 1, 2, \cdots$

- Complex exponential filtering:

$$\mathcal{H}_\alpha(z) = \frac{1}{1 - \alpha\, z^{-1}}$$

where $\alpha = \eta\, e^{j\theta}$, $\eta \in (0, 1)$

Examples of SCFs

- Output variances:

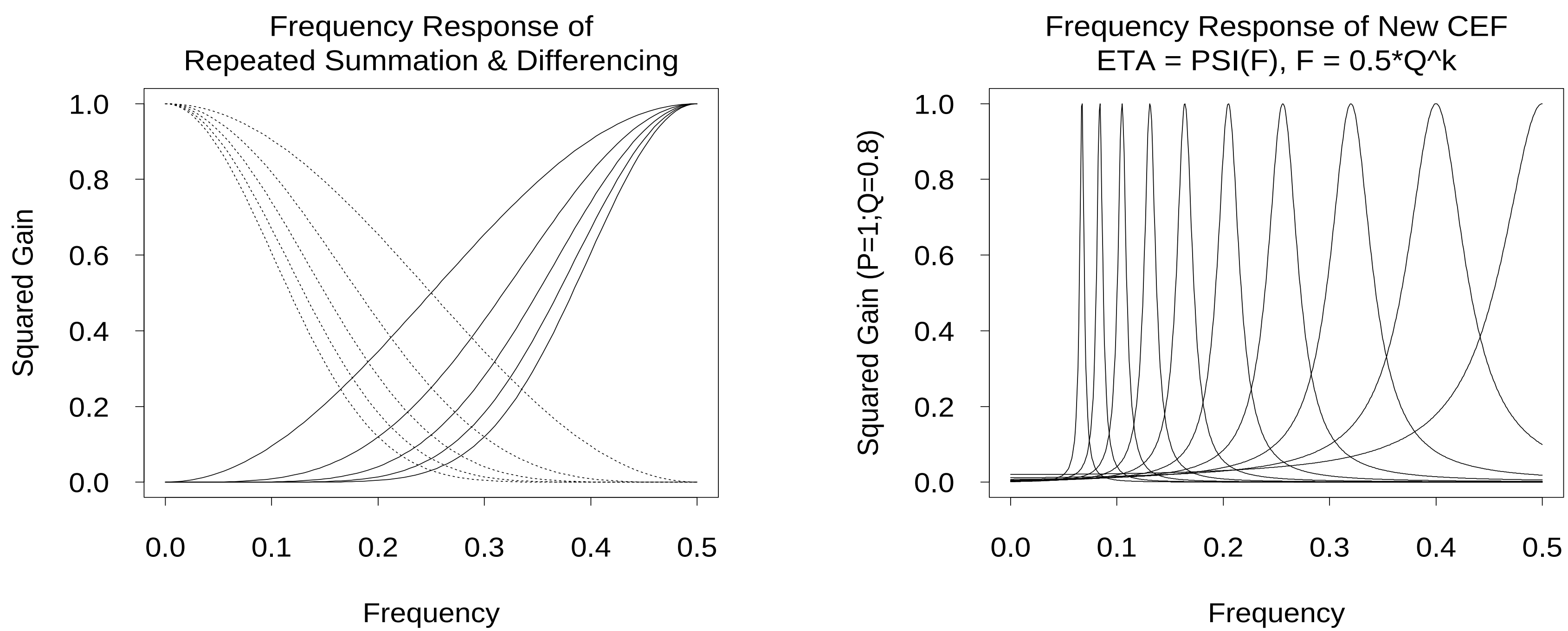
$$\sigma_\beta^2[n] = \mathrm{Var}\{Y_\beta[n]\}$$

- Output lag-one autocorrelation coefficients:

$$\gamma_\beta[n] = \mathrm{Corr}\{Y_\beta[n], Y_\beta[n-1]\}$$

- Output zero-crossing rates:

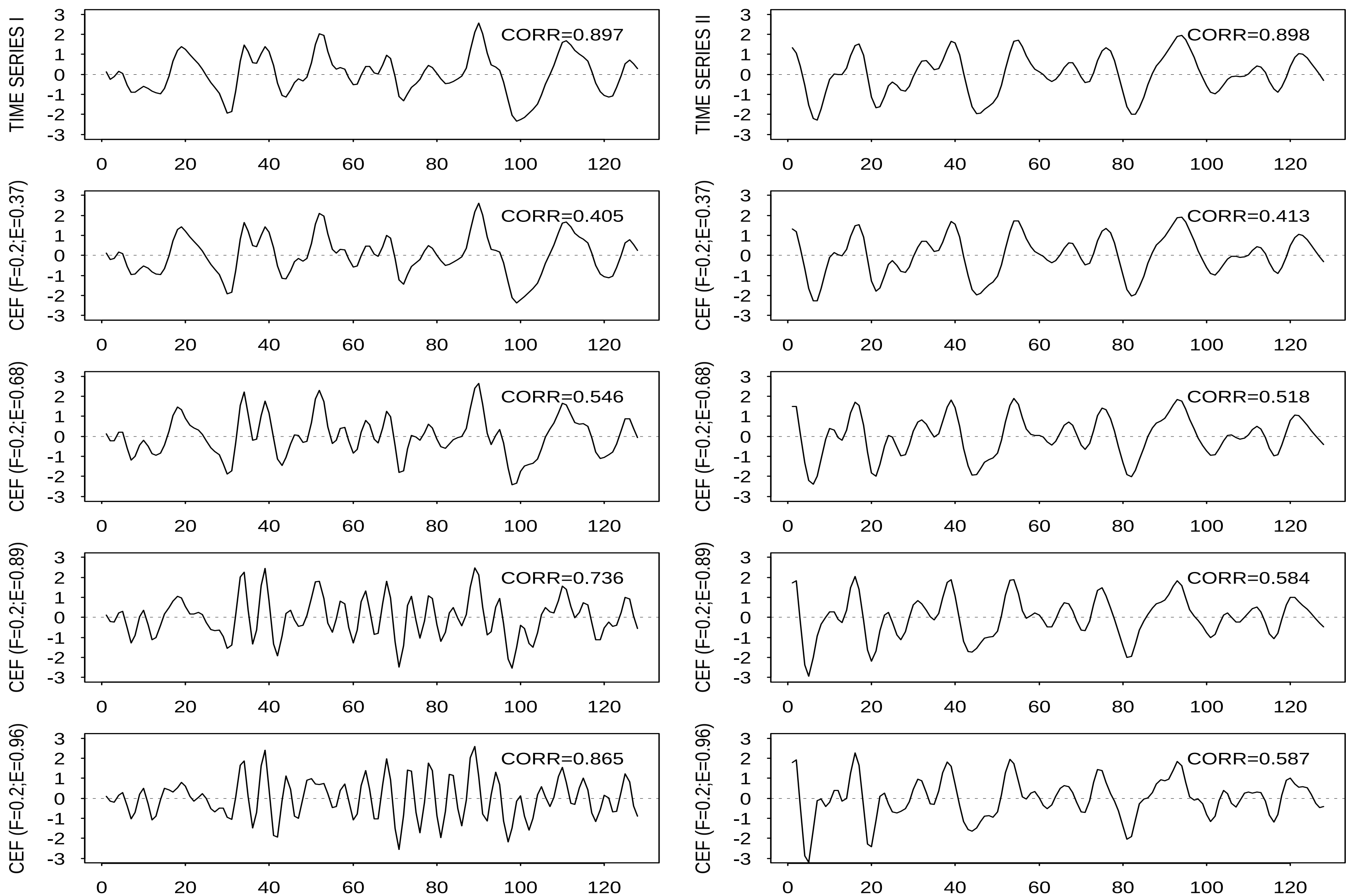
$$\zeta_\beta[n] = \mathrm{Pr}\{Y_\beta[n]\, Y_\beta[n-1] < 0\}$$


Frequency Response of
Repeated Summation & Differencing
Squared Gain
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
Frequency
Frequency Response of New CEF
ETA = PSI(F), F = 0.5*Q^k
Squared Gain (P=1;Q=0.8)
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
Frequency

WHAT DOES AN SCF DO?

- Data Reduction and Compression:
  - Summarize important features of filter output
- Signal Robustification and Enhancement:
  - Reject statistical variations of stochastic signals
  - Reject interference and noise via combination with an STFA filter bank

STFA FILTER BANK AND SCF AT WORK



INTERPRETATIONS OF OUTPUT ZCR'S

- Measure of oscillation patterns

- Measure of spectral mass center

- Equivalence to power spectral density if coupled with a suitable STFA filter bank

A CLASS OF NONSTATIONARY PROCESSES

- Nonstationary model:

$$Y(t) = X(g(t))$$

- Motivations: for returned Doppler signals,

$$g(t) = t - \frac{2}{c}\left(r_0 - \int_0^t v(\tau)\,d\tau\right)$$

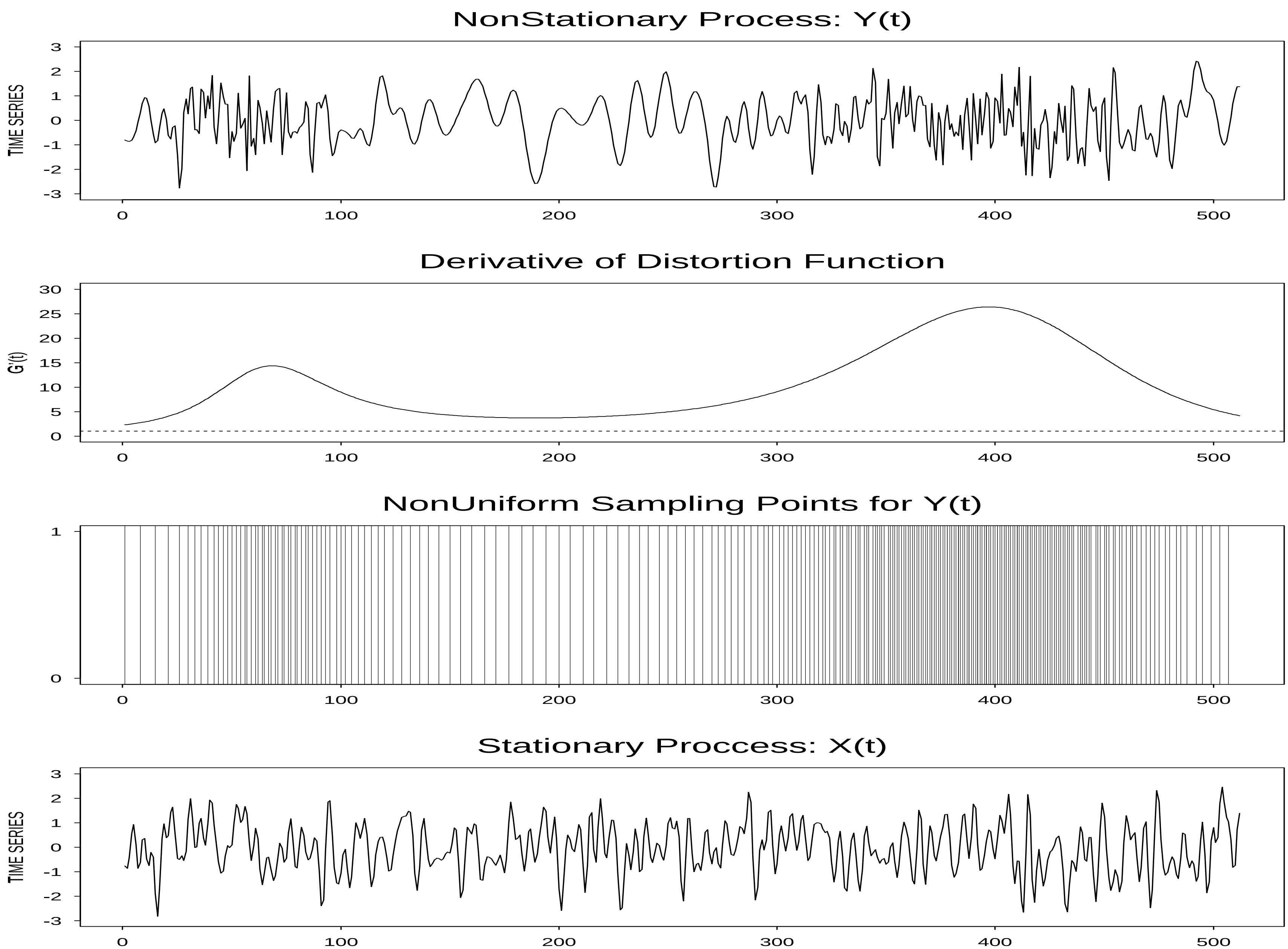
c: velocity of wave propagation

r_0: initial distance between receiver and target

$v(t)$: instantaneous velocity of target

BASIC ASSUMPTIONS

- $X(t)$ is a zero-mean stationary random process
- $g(t)$ is a "smooth" and monotone function which maps the interval $[0, T]$ onto itself
- Call $g(t)$ the distortion function (DF)


NonStationary Process: Y(t)
TIME SERIES
Derivative of Distortion Function
G'(t)
NonUniform Sampling Points for Y(t)
Stationary Proccess: X(t)
TIME SERIES

WHAT DOES A DF DO?

- Instantaneous ZCR inflation factor:

$$\zeta(t) = \dot{g}(t)\, c_0$$

where c_0 is the ZCR of $X(t)$

- Instantaneous bandwidth inflation factor:

$$\Omega(t) = \dot{g}(t)\, \Omega_0$$

where Ω_0 is the bandwidth of $X(t)$

HOW TO UNDO THE DISTORTION?

- Distortion function estimation:

$$\widehat{g}(t) = T \frac{\int_0^t \widehat{\mu}(\tau)\, d\tau}{\int_0^T \widehat{\mu}(\tau)\, d\tau}$$

where $\widehat{\mu}(t)$ is an estimator of $\mu(t) \propto \dot{g}(t)$

- Inverse transformation:

$$\widehat{X}(t) = Y\left(\widehat{g}^{-1}(t)\right)$$

STATISTICAL TIME-FREQUENCY ANALYSIS

- Choice of filter bank: repeated differencing (RD)

- Choice of SCF: ZCR's from the RD filter bank

- Results of STFA: $\zeta_k(t) = \dot{g}(t)\, c_k \quad (k = 0, 1)$

  - $\{\zeta_0(t), \zeta_1(t)\}$: ZCR's of $Y(t)$ and $\dot{Y}(t)$

  - $\{c_0, c_1\}$: ZCR's of $X(t)$ and $\dot{X}(t)$

NONPARAMETRIC DF ESTIMATION

- Sampling: $Y[n] = Y(n\Delta)$

- Filtering: $Y_k[n] = \mathcal{H}_k(z)\, Y[n], \quad (k = 0, 1)$

- Instantaneous SCF:

$$Z_k[n] = \mathcal{I}\{Y_k[n]\, Y_k[n-1] < 0\} \in \{0, 1\}$$

NONPARAMETRIC DF ESTIMATION (CONT'D)

- Regression setting:

$$Z_k[n] = \mu_k(n\Delta) + \epsilon_k[n]$$

where

$$\mu_k(n\Delta) = E\{Z_k[n]\} = \dot{g}(n\Delta)\, c_k\, \Delta$$

- Nonparametric smoothing:

$$\widehat{\mu}_k(n\Delta) = \text{Smooth}\{Z_k[n]\}$$

SIGNAL ENHANCEMENT BY STFA

- SNR in the regression problems:

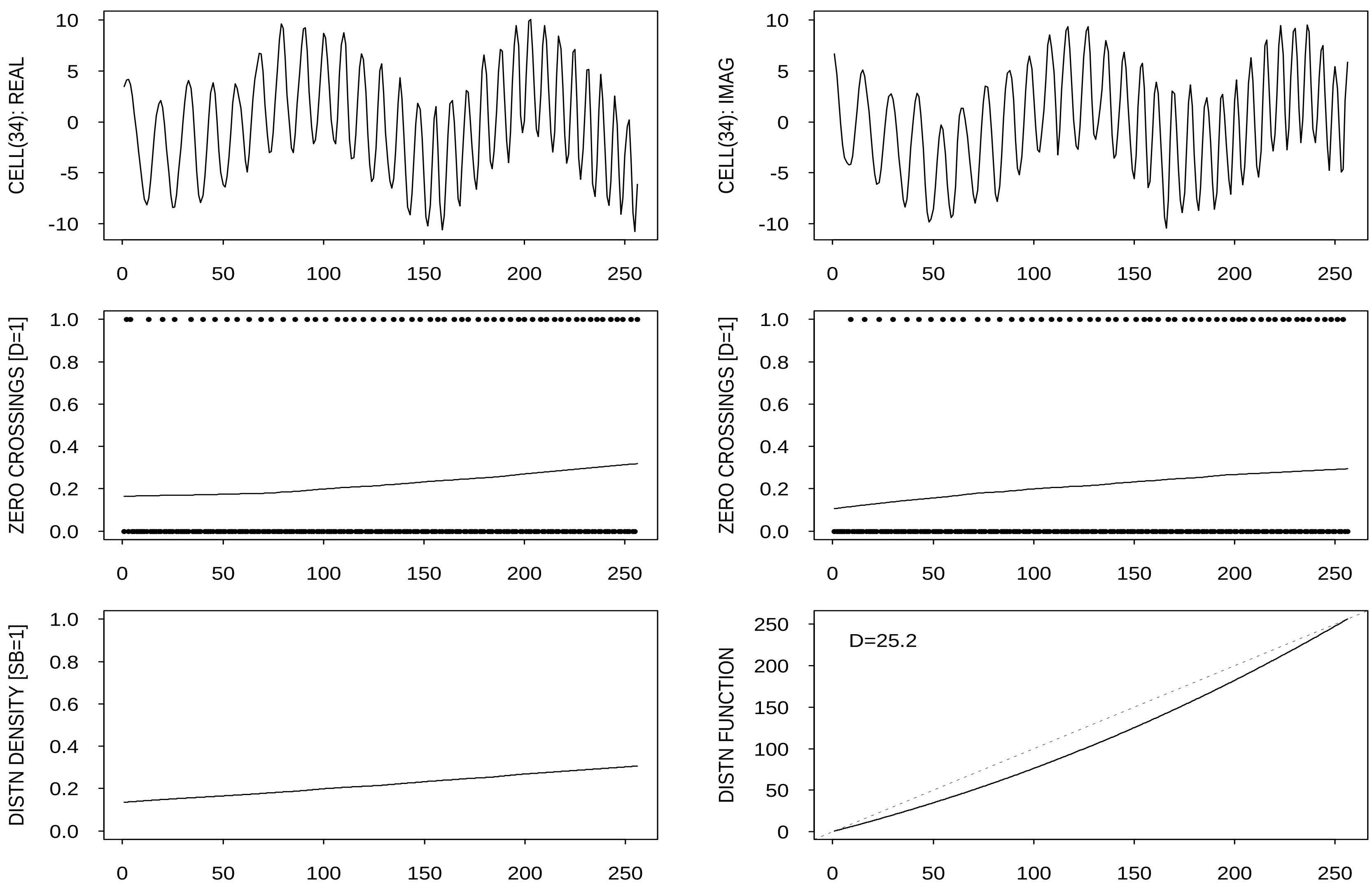
$$\text{SNR}_k(n\Delta) = \frac{(E\{Z_k[n]\})^2}{\text{Var}\{Z_k[n]\}} = \frac{\mu_k(n\Delta)}{1 - \mu_k(n\Delta)}$$

- Implications:

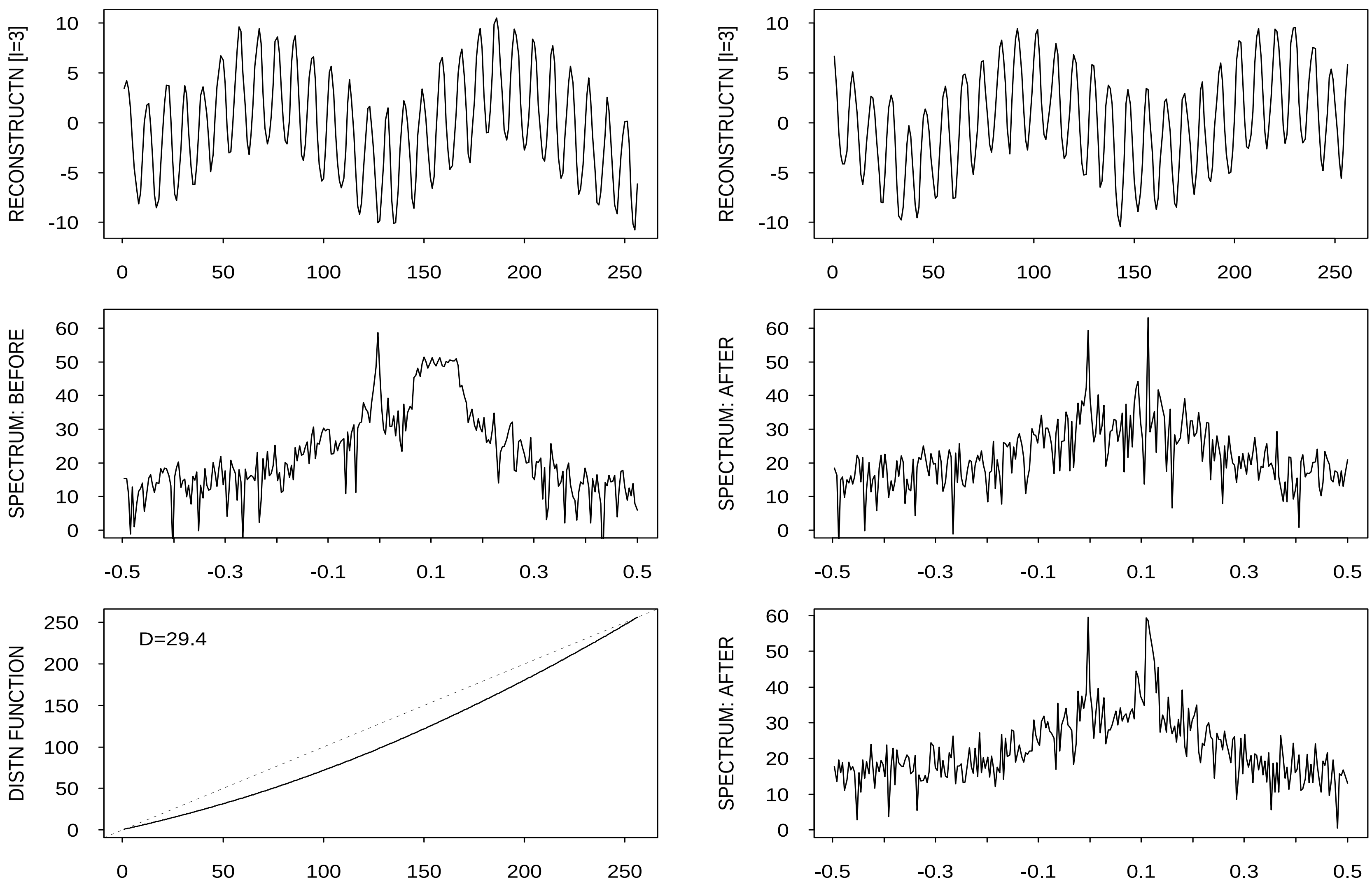
$$\mu(t)\uparrow \iff \text{SNR}(t)\uparrow$$

$$\text{SNR}_1(t) > \text{SNR}_0(t) \iff c_1 > c_0$$

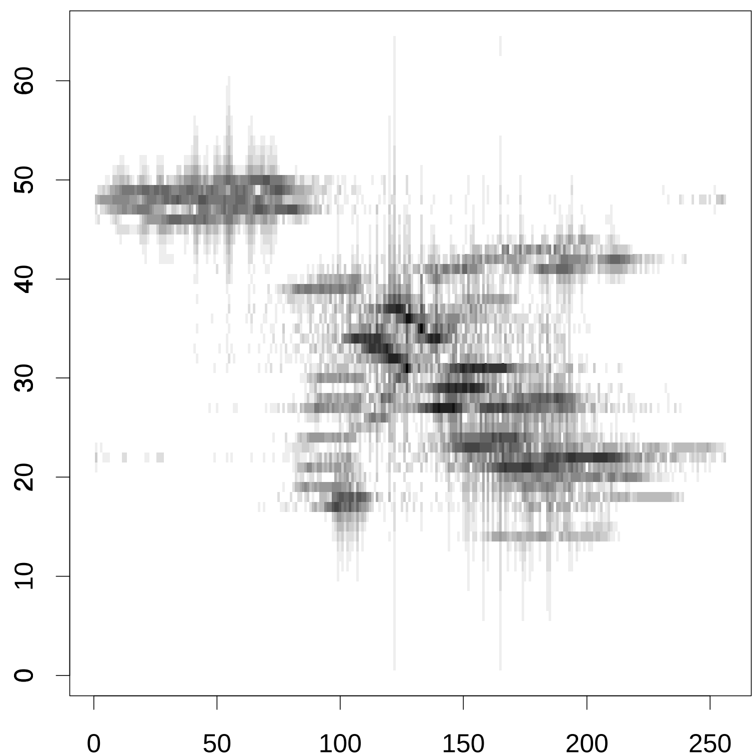
EXAMPLE OF DISTORTION CORRECTION



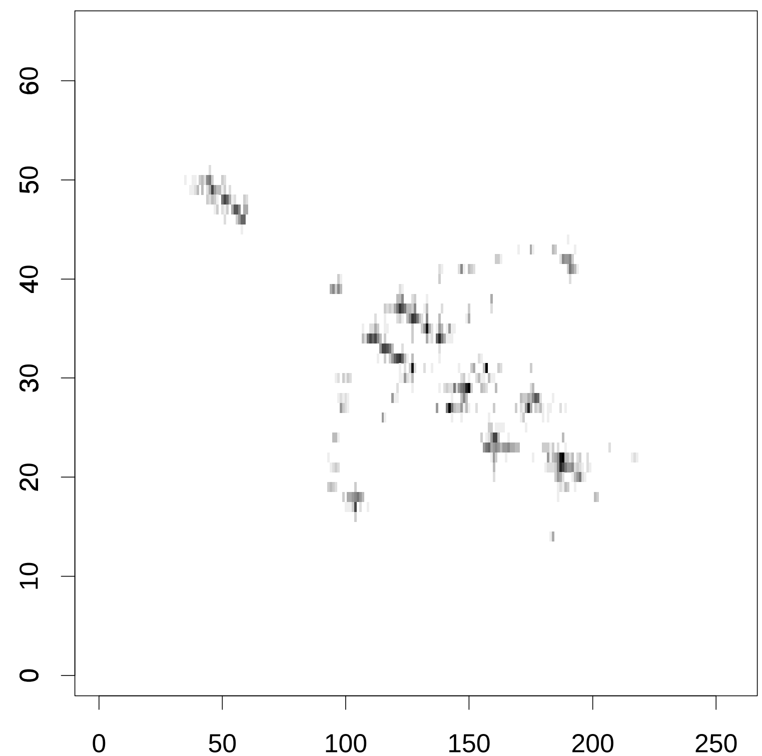
EXAMPLE OF DISTORTION CORRECTION (CONT'D)



ISAR IMAGE OF B727S



ISAR IMAGE OF B727S (DEBLURRED)



CONCLUSIONS

- Statistical time-frequency analysis framework
- Combination of STFA filter banks with statistical characterization functions (SCF’s)
- Data reduction and compression, signal robustification and enhancement