

MODELLING AND ANALYZING BREAKDOWN PHENOMENA
IN INSULATORS - A STOCHASTIC APPROACH

Research Thesis

Submitted in partial fulfillment of the requirements
for the degree of Doctor of Science

ENMANUEL YASHCHIN

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