



Multidimensional Blocking in UPC

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Overview

- **Current data distribution in UPC**
- **Multiblock extension to UPC**
- **Locality analysis**
- **Results**
- **Conclusions and Future Work**

Unified Parallel C

- **Parallel extension to C**
 - Allow the declaration of shared variables
 - Programmer specifies shared data distribution
 - Work-sharing construct used to distribute loop iterations
 - Synchronization primitives: barriers, fences, and locks
- **Data can be private, shared local and shared remote**
 - Shared data can be accessed by all threads
 - Shared space is divided into portions with affinity to a thread
 - Latency to access *local* shared data is usually lower than latency to access *remote* shared data
- **All threads execute the same program (SPMD style)**

Data Distributions in UPC

- **Data distributions are an essential part of a PGAS language**
- **Affinity directives are already in the language**
 - shared [B] int A[N]
- **This distribution is not particularly well-suited for applications in specific domains**
 - Does not allow easy interface with existing high performance libraries

Multidimensional Blocking in UPC

- **Extend shared data layout directive to support multidimensional distributions**
- **Programmer specifies n-dimensional *tiles* for shared data**

```
shared [2] [2] int X[10][10];
```

New Blocking factor

Array size

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	1	17	18	1
2	21	22	23	24	25	26	27	28	29
0	30	31	32	33	34	35	3	37	3

```
#define BLK_SIZE (sizeof(double)*b*b)
shared [b][b] double A[M][P], B[P][N], C[M][N];

upc_forall(int ii = 0; ii < M; ii += BS; continue)
  upc_forall(int jj = 0; jj < N; jj += BS; &C[ii][jj]) {
    double scratchA[b][b], scratchB[b][b];

    upc_memget(scratchA, &A[ii][jj], BLK_SIZE);
    upc_memget(scratchB, &B[ii][jj], BLK_SIZE);

    dgemm(&C[ii][jj], &scratchA, &scratchB, b, b, b);
  }
```

Drawbacks to Multidimensional Blocking

- **Accessing shared data becomes more expensive**
 - Computing indexes requires sum and products across each shared dimension
- **Compiler can identify accesses to *local* shared data and privatize them**
 - Remove the overhead of calls to the runtime library
 - Expose the indexing computations to the compiler so they can be optimized

Locality Analysis

Goal

Compute symbolically the node ID of each shared reference in the `upc_forall` loop and compare it to the node ID of the affinity expression.

`upc_forall (int i=0; i < N; i++; &A[i])`

- **Shared reference inside the loop has a displacement with respect to the affinity expression**
 - Specified by the distance vector $\mathbf{k} = [k_0, k_1, \dots, k_{n-1}]$

Locality Analysis

- **Observation 1**
 - A block shifted by a distance vector \mathbf{k} can span *at most* two threads in each dimension
 - Locality can change in only one place in this dimension, which we call the *cut*
- **Observation 2**
 - Only need to test the locality at the 2^n corners of the block
 - All elements from the *corner* to the *cut* will have the same owner

Identifying Local Shared Accesses

- **For a given upc_forall loop**
 - Split the loop into multiple iteration spaces where the shared reference is either local or remote
 - Use cuts to determine split points
 - For each iteration space, keep a *position* relative to the affinity test
 - Local shared references are privatized
 - Remote shared references are transformed into calls to the UPC Runtime

Computing Cuts

```
/*8 threads and 2 threads/node */
shared [2][3] int A[8][9];
```

```
for(v0=0; v0 <7 ; v0++) {
    upc_forall(v1=0; v1<6; v1++; &A[v0][v1]){
        A[v0+1][v1+2] = v0*v1;
    }
}
```

k =
[1,2]

	0	1	2	3	4	5	6	7	8
0	T ₀	T ₀	T ₀	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂
1	T ₀	T ₀	T ₀	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂
2	T ₃	T ₃	T ₃	T ₄	T ₄	T ₄	T ₅	T ₅	T ₅
3	T ₃	T ₃	T ₃	T ₄	T ₄	T ₄	T ₅	T ₅	T ₅
4	T ₆	T ₆	T ₆	T ₇	T ₇	T ₇	T ₀	T ₀	T ₀
5	T ₆	T ₆	T ₆	T ₇	T ₇	T ₇	T ₀	T ₀	T ₀
6	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂	T ₃	T ₃	T ₃
7	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂	T ₃	T ₃	T ₃

Computing Cuts

```
/*8 threads and 2 threads/node */
shared [2][3] int A[8][9];
```

```
for(v0=0; v0 <7 ; v0++) {
  upc_forall(v1=0; v1<6; v1++; &A[v0][v1]) {
    A[v0+1][v1+2] = v0*v1;
  }
}
```

k =
[1,2]

	0	1	2	3	4	5	6	7	8
0	T ₀	T ₀	T ₀	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂
1	T ₀	0	T ₀	T ₁	0	T ₁	T ₂	1	T ₂
2	T ₃	T ₃	T ₃	T ₄	T ₄	T ₄	T ₅	T ₅	T ₅
3	T ₃	1	T ₃	T ₄	2	T ₄	T ₅	2	T ₅
4	T ₆	T ₆	T ₆	T ₇	T ₇	T ₇	T ₀	T ₀	T ₀
5	T ₆	3	T ₆	T ₇	3	T ₇	T ₀	0	T ₀
6	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂	T ₃	T ₃	T ₃
7	T ₁	0	T ₁	T ₂	1	T ₂	T ₃	1	T ₃

Dimensions 0 to $n-2$

$$\text{Cut}_i = b_i - k_i \% b_i$$

$$\text{Cut}_0 = 2 - 1 \% 2$$

$$\text{Cut}_0 = 1$$

Computing Cuts

```
/*8 threads and 2 threads/node */
shared [2][3] int A[8][9];
```

k =

k' = [k₀+b₀-1, k₁]
 k' = [2,2]

```
for(v0=0; v0 <7 ; v0++) {
    upc_forall(v1=0; v1<6; v1++; &A[v0][v1]) {
        A[v0+1][v1+2] = v0*v1;
    }
}
```

	0	1	2	3	4	5	6	7	8
0	T ₀	T ₀	T ₀	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂
1	T ₀	0	T ₀	T ₁	0	T ₁	T ₂	1	T ₂
2	T ₃	T ₃	T ₃	T ₄	T ₄	T ₄	T ₅	T ₅	T ₅
3	T ₃	1	T ₃	T ₄	2	T ₄	T ₅	2	T ₅
4	T ₆	T ₆	T ₆	T ₇	T ₇	T ₇	T ₀	T ₀	T ₀
5	T ₆	3	T ₆	T ₇	3	T ₇	T ₀	0	T ₀
6	T ₁	T ₁	T ₁	T ₂	T ₂	T ₂	T ₃	T ₃	T ₃
7	T ₁	0	T ₁	T ₂	1	T ₂	T ₃	1	T ₃

Dimension n-1

$$\text{Cut}^{\text{Upper}} = (\text{tpn} - \text{course}(k)) \times b_{n-1} - k_{n-1} \% b_{n-1}$$

$$\text{Cut}^{\text{Upper}} = (2 - 0) \times 3 - 2 \% 3$$

$$\text{Cut}^{\text{Upper}} = 4$$

$$\text{Cut}^{\text{Lower}} = (\text{tpn} - \text{course}(k')) \times b_{n-1} - k_{n-1} \% b_{n-1}$$

$$\text{Cut}^{\text{Lower}} = (2 - 1) \times 3 - 2 \% 3$$

$$\text{Cut}^{\text{Lower}} = 1$$

8 UPC Threads
2 Nodes
4 Threads per node

Algorithm – Phase 1: Collection

```

1 shared [5][5] int A[20][20];
2 for (i=0; i < 19; i++)
3   upc_forall (j=0; j < 20; j++; &A[i][j]) {
4     A[i+1][j] = MYTHREAD;
5   }

```

- **Collect shared references in upc_forall loop**
- **Compute distance vector**

$A[i+1][j] = MYTHREAD; \quad [1,0]$

	0	1	2	3	4	5	...
0	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
1	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
2	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
3	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
4	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
5	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅
...	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅

Phase 2: Restructure Loop

```

1 shared [5][5] int A[20][20];
2 for (i=0; i < 19; i++)
3   upc_forall (j=0; j < 20; j++; &A[i][j]) {
4     A[i+1][j] = MYTHREAD;
5   }

```

8 UPC Threads

2 Nodes

4 Threads per node

Distance Vector:

[1,0]

- **Starting at outer loop, iteratively compute cuts for each loop**

- **Outer Loop:**

$$\text{Cut}_i = b_i - k_i \% b_i$$

$$\text{Cut}_0 = 4$$

	0	1	2	3	4	5	...
0	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
1	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
2	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
3	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
4	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
5	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅
...	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅

Phase 2: Restructure Loop

```

1 shared [5][5] int A[20][20];
2 for (i=0; i < 19; i++)
3   if ( (i%5) < 4) {
4     upc_forall (j=0; j < 20; j++; &A[i][j]) {
5       A[i+1][j] = MYTHREAD;
6     }
7   }
8   else {
9     upc_forall (j=0; j < 20; j++; &A[i][j]) {
10      A[i+1][j] = MYTHREAD;
11    }
12  }

```

	0	1	2	3	4	5	...
0	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
1	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
2	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
3	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
4	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
5	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅
...	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅

8 UPC Threads

2 Nodes

4 Threads per node

Distance Vector:

[1,0]

- **Inner Loops:**

- Offset is 0 so $\text{Cut}^{\text{Upper}}$ and $\text{Cut}^{\text{Lower}}$ are 0

Phase 3: Privatize Local Accesses

8 UPC Threads
2 Nodes
4 Threads per node

Distance Vector:

```

1 shared [5][5] int A[20][20];
2 for (i=0; i < 19; i++)
3   if ( (i%5) < 4) {
4     upc_forall (j=0; j < 20; j++; &A[i][j]) {
5       A[i+1][j] = MYTHREAD;
6     }
7   }
8   else {
9     upc_forall (j=0; j < 20; j++; &A[i][j]) {
10      A[i+1][j] = MYTHREAD;
11    }
12  }

```

[1, 0]
Region Position: [0,0]
Reference position: [1,0]
Node ID: 0 (Local)

Region Position: [4,0]
Reference Position: [5,0]
Node ID: 1 (Remote)

	0	1	2	3	4	5	...
0	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
1	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
2	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
3	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
4	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
5	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅
...	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅

Phase 3: Privatize Local Accesses

```

1 shared [5][5] int A[20][20];
2 for (i=0; i < 19; i++)
3   if ( (i%5) < 4) {
4     upc_forall (j=0; j < 20; j++; &A[i][j]) {
5       offset = ComputeOffset(i,j);
6       *(base_A+offset) = MYTHREAD;
7     }
9   }
9   else {
10    upc_forall (j=0; j < 20; j++; &A[i][j]) {
11      A[i+1][j] = MYTHREAD;
12    }
13  }

```

8 UPC Threads

2 Nodes

4 Threads per node

Distance Vector:

[1,0]

	0	1	2	3	4	5	...
0	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
1	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
2	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
3	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
4	T ₀	T ₀	T ₀	T ₀	T ₀	T ₁	T ₁
5	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅
...	T ₄	T ₄	T ₄	T ₄	T ₄	T ₅	T ₅

Preliminary Results

- **Machine**

- 4 nodes of an development IBM Squadron Cluster
 - Each node: 8 SMP POWER5 processors, 1.9GHz; 16GB of memory

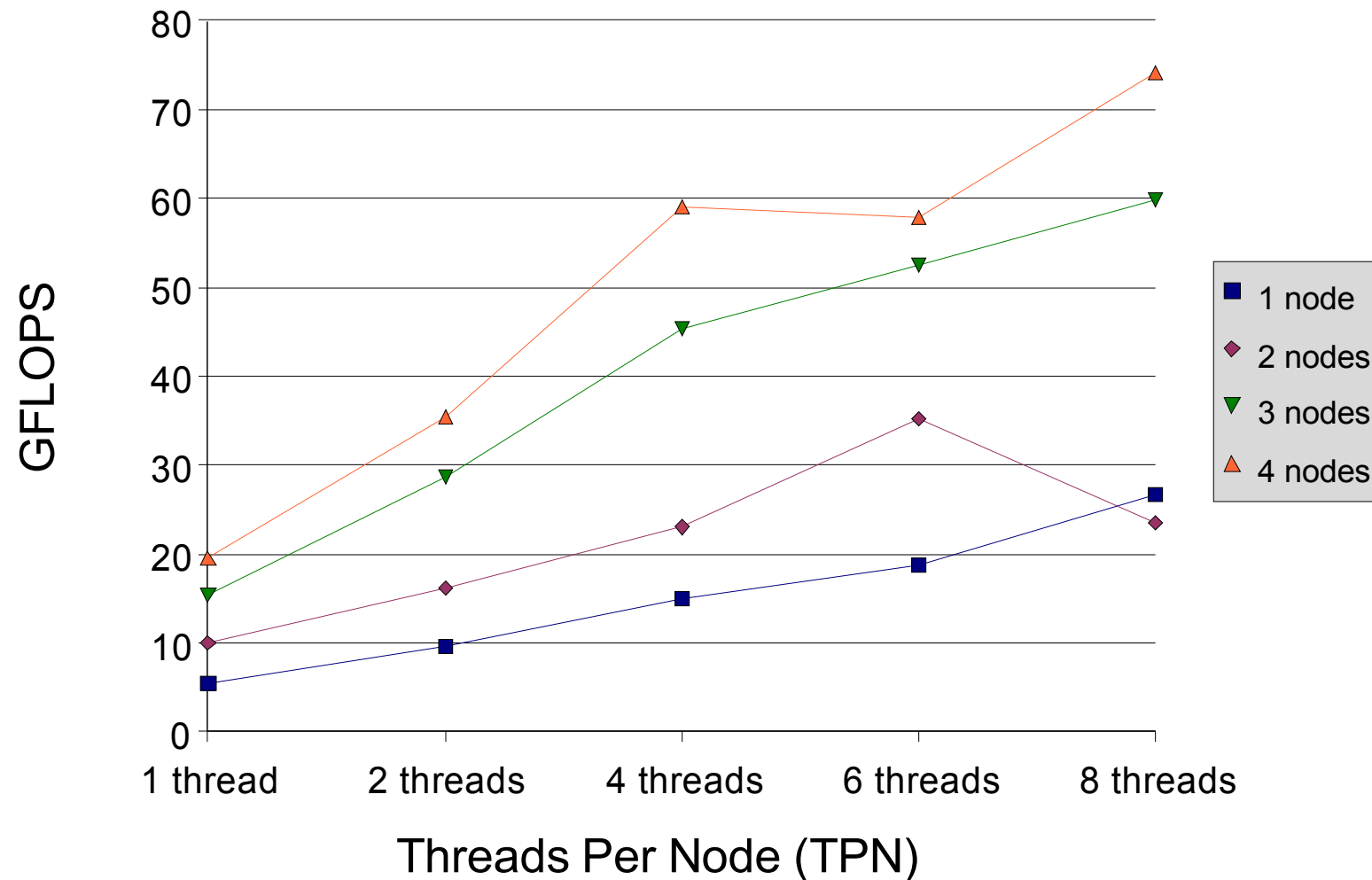
- **Benchmarks**

- Cholesky factorization and Matrix Multiply
 - Showcase multiblocking for shared arrays
 - Implementation uses ESSL library for multiplying matrices
- Dense matrix-vector multiply and 5-point stencil
 - Demonstrate performance impact of locality analysis and privatization

Cholesky Factorization

Theoretical Peak: $6.9\text{GFlops} \times \text{TPN} \times \text{nodes}$

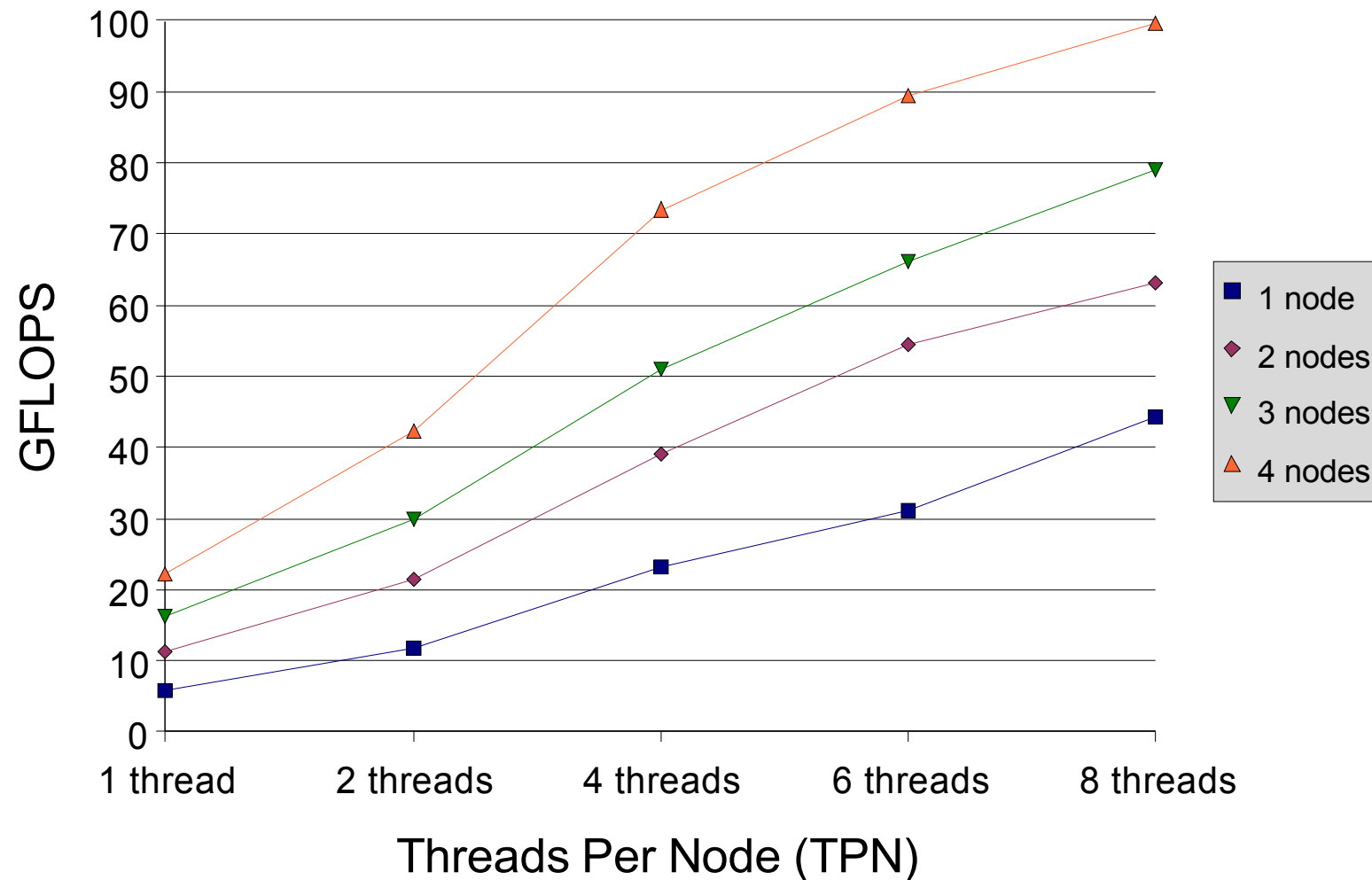
Cholesky Performance (GFlops)



Matrix Multiplication

Theoretical Peak: $6.9\text{GFlops} \times \text{TPN} \times \text{nodes}$

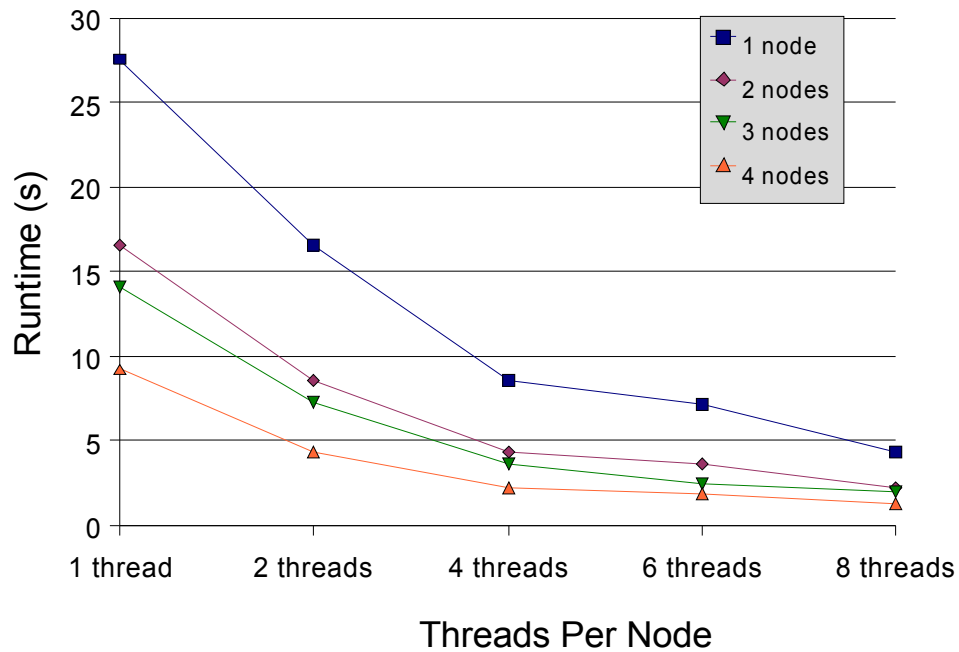
Matrix Multiplication Performance (GFlops)



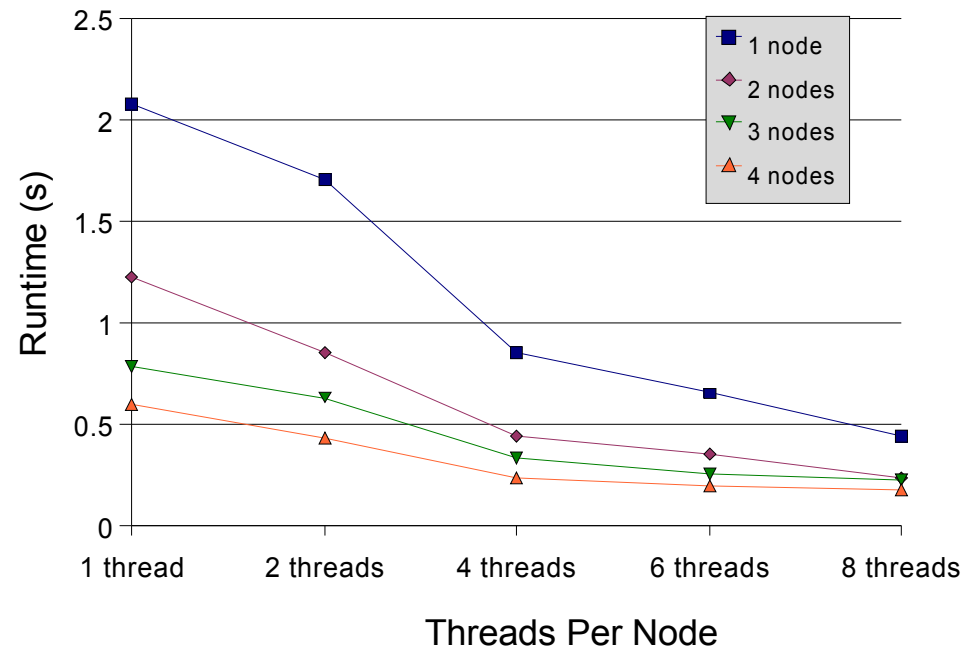
Locality Analysis and Privatization

Sequential C Time: 0.47s

Non-Optimized Matrix-Vector Multiplication



Optimized Matrix-Vector Multiplication

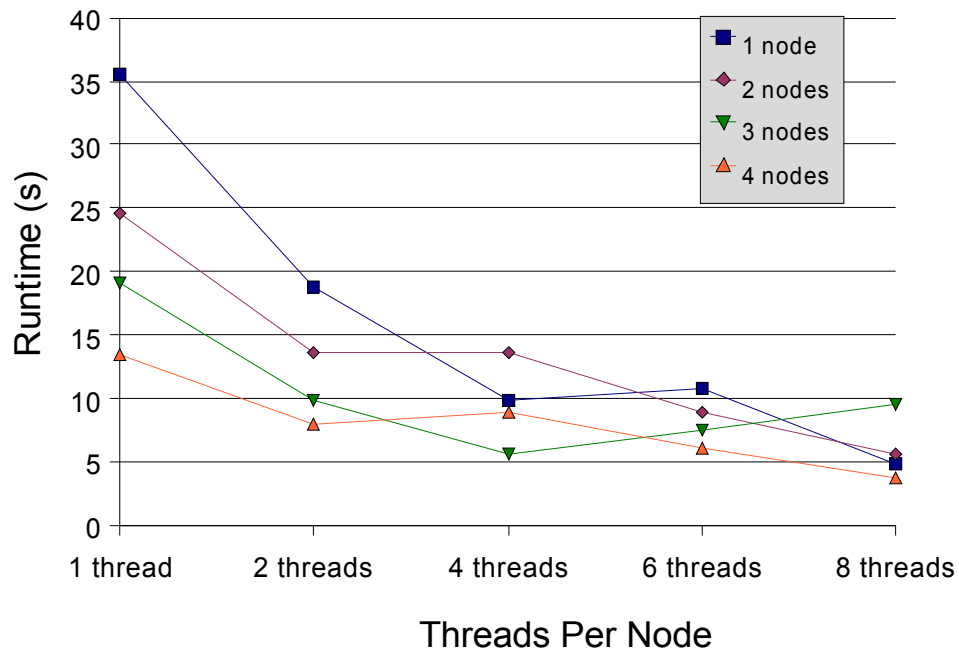


```
shared [*] double A[14400][14400];
```

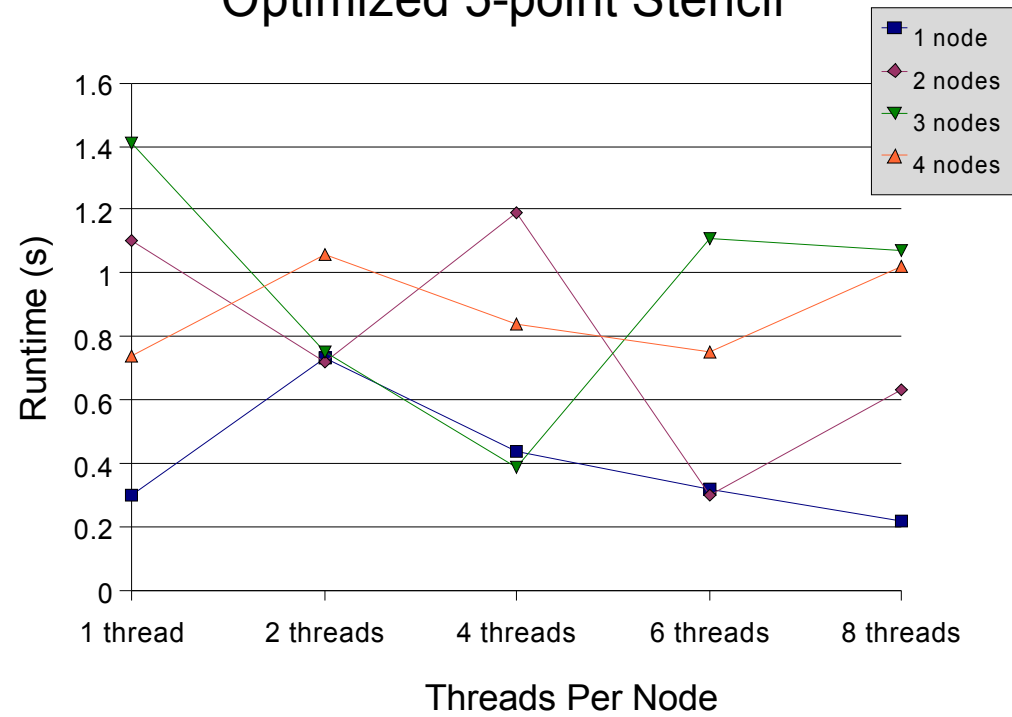
Locality Analysis and Privatization

Sequential C Time: 0.17s

Non-Optimized 5-point Stencil



Optimized 5-point Stencil



```
shared [100][100] double A[6000][6000];
```

Conclusions

- **Multiblocked arrays fine control over data layout**
 - Allows programmer to obtain better performance while simplifying expression of computations
 - Provides ability to integrate with existing libraries (C and Fortran) that require specific memory layouts
- **Locality analysis**
 - Compiler is able to reduce the overheads introduced by accessing shared arrays

Future Work

- **Multiblocked arrays**
 - Add processor tiling to increase programmer's ability to write code that scales to a large number of processors
 - Define a set of collectives that are optimized for the UPC programming model that will address scalability issues
- **Compiler**
 - Reduce overhead of computing indexes of privatized accesses
 - Investigate how traditional loop optimizations can be used with `upc_forall` loops to further improve performance

Questions