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Stochasic Unit Committment Problem

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Abstract

The unit commitment problem is to find the production scheduling of a set of electric power generating units, and the generation levels for each unit, over a short term, typically from 24 hours to one week. The objective is to find the optimal schedule which meets the energy demand forecast at a minimum cost, and satisfy technological constraints such as the minimum up and down times for the units. First, we assume the demands are known and made a bench mark for the mixed-integer programming formulations for this model. Second, we consider the case with uncertain demands and review stochastic versions of the problem, which allow the schedule to adjust to the observed demands. Then, we tested two different Lagrangian relaxations techniques for solving the stochastic programming problem. Finally, we tested a heuristic method we developed, based on the linear relaxation for some binary variables, which exploits the structure of the stochastic problem.

1 Definition of the Unit Commitment Problem

The Unit commitment problem can be stated as finding the optimal scheduling of production of electric power generating units over a short term, typically from 24 hours to one week, in order to minimize the operations costs. This optimal solution must satisfy the operating constraints and must satisfy the demand forecast. Then, following the definition, and the models proposed in [1, 4], the base deterministic mathematical model can be stated as,

$$\text{Minimize: } \sum_{i=1}^{I}\sum_{t=1}^{T} z_{i,t}F_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_{i,t}C_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} y_{i,t}S_{it} \tag{1}$$

$$\text{Subject to: } \sum_{i}^{I} g_{i,t} \geq d_t,\ t = 1, \ldots, T, \tag{2}$$

$$g_{i,t} \leq Q_i z_{i,t},\ t = 1, \ldots, T, i = 1, \ldots, I \tag{3}$$

$$g_{i,t} \geq q_i z_{i,t},\ t = 1, \ldots, T, i = 1, \ldots, I \tag{4}$$

$$y_{i,t} \geq z_{i,t} - z_{i,t-1},\ t = 2, \ldots, T, i = 1, \ldots, I \tag{5}$$

$$z_{i,t} - z_{i,t-1} \leq z_{i,\tau}, \tau = t, \ldots, \min\{t + L_i - 1, T\}, t = 2, \ldots, T, i = 1, \ldots, I \tag{6}$$

$$z_{i,t-1} - z_{i,t} \leq 1 - z_{i,\tau}, \tau = t, \ldots, \min\{t + l_i - 1, T\}, t = 2, \ldots, T, i = 1, \ldots, I \tag{7}$$

$$z_{i,t}, y_{i,t} \in \{0, 1\} \tag{8}$$

$$g_{i,t} \geq 0, \tag{9}$$

where we have the following definitions:

- $g_{i,t}$ is a continuous variable that represents the MV of energy produced by generator i in period t,

- $y_{i,t}$ is a binary variable that is 1 if generator i is started at the beginning of period t, 0 otherwise,
- $z_{i,t}$ is a binary variable that is 1 if generator i is on during period t, 0 otherwise,
- $F_{i,t}$ is the fixed cost in \$/period of operating generator i in period t,
- $C_{i,t}$ is the cost of generation for generator i in period t in \$/MV/period,
- $S_{i,t}$ is the cost of start up for generator i in period t in \$,
- Q_i is the upper bound in MV for energy generated with generator i in every period,
- q_i is the lower bound in MV for energy generated with generator i in every period,
- L_i is the minimum up time for generator i when it is started up,
- l_i is the minimum down time for generator i when it is shutted down,
- d_t the energy demand in period t,
- $1, \ldots, I$ is the set of generators,
- and $1, \ldots, T$ is the set of time periods.

Then, given a set $\{i = 1, \ldots, I\}$ of generators which can be of different types, and a set of time periods $\{t = 1, \ldots, T\}$, the objective function (1) minimize the total operative costs. That is, the sum of the fixed operating cost $F_{i,t}$, the generating costs $C_{i,t}$, and the start up costs $S_{i,t}$, for all the generators in all time slots. In constraint (2) we have that the energy produced by all generators must meet or exceed the demand d_t in each period. Now, in constraints (3) and (4) we have that the generators must operate within their minimum an maximum generations bounds. Equation (5) is controlling when the generators are started up. Finally, in equations (6) and (7) we have that the generators must accomplish the minimum up and down times.

In this work we analyzed the stochastic unit commitment problem, where uncertainty on the demand forecast is considered as a new element in the base model presented in (1)-(9). Then, the goal in this project is to analyze the performance of the current methods for solving the stochastic unit commitment problem as well as explore a new method to find feasible solutions close to the optimal in short times. We explore an heuristic algorithm based in linear relaxations of some of the binary variables of the base model, which exploits the structure of the stochastic problem. This heuristic showed a competitive performance both in time and in the quality of the solutions in the experiments.

The structure of this paper is as follows. In Section 2 we present a short review on some improved formulations for the deterministic unit commitment problem. In Section 3 we review two stochastic formulations. In Section 4 we review some solutions methods for the stochastic formulations. Then, in Section **??** we explain the heuristic "LPRolling" proposed in this paper. Finally, in Section **??** conclusions are drawn.

2 Formulations review

Several efforts have been done trying to improve the formulation of the unit commitment problem for having a more efficient way finding the schedule. Rajan and Takriti developed a tighter formulation for the problem in [6] working over the minimum up and down constraints. In this work they presented a set of inequalities that define the convex hull for the set defined by the minimum up and down constraints (6), (7), in the unit commitment problem formulation. Additionally, Carrion and Arroyo presented in [3] an alternative formulation where they reduce the number of binary variables as well as the number of constraints. Then, using these two formulations and the formulation (1)-(9) for solving some instances of the unit commitment problem we could find that the formulation proposed by Rajan and Takriti was generally more efficient. For example, for some instances of 32 units and 72 periods and using CPLEX 11.0 to solve the problem we had the results shown in Table 1.

Therefore, for the stochastic version of the unit commitment problem we used the formulation proposed in [6].

Instance	Formulation	LP time	Total time	Number of B&B nodes
1	General	15.31	436	150
	Carrion & Arroyo	0.22	36	100
	Rajan & Takriti	0.13	1	0
2	General	14.66	314	10
	Carrion & Arroyo	0.15	46	177
	Rajan & Takriti	0.12	1	0
3	General	11.71	2240	510
	Carrion & Arroyo	0.18	242	700
	Rajan & Takriti	0.11	62	400
4	General	1.98	3290	590
	Carrion & Arroyo	0.06	165	300
	Rajan & Takriti	0.04	46	300

The 32 units, 72 periods instances were solved with CPLEX 11.0 with a gap of 0.05%

Table 1: Formulations comparison

3 Stochastic Models

In 1968 Muckstadt and Wilson in [5] presented the first model that incorporate the randomness in the energy demands. Takriti et al. formally presented the first stochastic model in [7]. This was a multistage model formulated as follows,

$$\text{Minimize: } \sum_{s\in S} P_s \left(\sum_{i=1}^{I}\sum_{t=1}^{T} z_{i,t}^s F_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_{i,t}^s C_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} y_{i,t}^s S_{it} \right) \tag{10}$$

$$\text{Subject to: } \sum_{i}^{I} g_{i,t}^s \geq D_t^s,\ t = 1,\dots,T, s\in S \tag{11}$$

$$g_{i,t}^s \leq Q_i z_{i,t}^s,\ t = 1,\dots,T, i = 1,\dots,I, s\in S \tag{12}$$

$$g_{i,t}^s \geq q_i z_{i,t}^s,\ t = 1,\dots,T, i = 1,\dots,I, s\in S \tag{13}$$

$$y_{i,t}^s \geq z_{i,t}^s - z_{i,t-1}^s,\ t = 2,\dots,T, i = 1,\dots,I \tag{14}$$

$$z_{i,t}^s - z_{i,t-1}^s \leq z_{i,\tau}^s, \tau = t,\dots,\min\{t+L_i-1,T\}, t = 2,\dots,T, i = 1,\dots,I, s\in S \tag{15}$$

$$z_{i,t-1}^s - z_{i,t}^s \leq 1 - z_{i,\tau}^s, \tau = t,\dots,\min\{t+l_i-1,T\}, t = 2,\dots,T, i = 1,\dots,I, s\in S \tag{16}$$

$$z_{i,t}^s, y_{i,t}^s \in \{0,1\}\ t = 1,\dots,T, i = 1,\dots,I, s\in S \tag{17}$$

$$g_{i,t}^s \geq 0,\ t = 1,\dots,T, i = 1,\dots,I, s\in S \tag{18}$$

where constraints (11) - (18) must be satisfied for all possible scenarios $s\in S$, and the superscript s identifies to which scenario belongs each variable.

Additionally, for consistency in the solution a set of constraints called Bundle constraints (nonaticipativity) are added to the formulation. That is, if two scenarios s and s' are indistinguishable up to time t, as illustrated in Figure 1, then the decisions for both scenarios by time t might be the same. Therefore, a Bundle Ω_k is a set of scenarios that up to time t have the same demands for every period in $1,\dots,t$. Now, the Bundle of s at time t is denoted by $B(s,t)$. Hence, if scenario s belongs to bundle Ω_k at time t we say that $B(s,t) = \Omega_k$. As a result, if two scenarios s_1, s_2 are members of the same bundle at time t, then we have

$$B(s_1,t) = B(s_2,t) = \Omega_k \Rightarrow B(s_1,\tau) = B(s_2,\tau), \tau = 1,\dots,t-1.$$

Hence, the following constraints are added to the problem,

$$B(s_1,t) = B(s_2,t) = \Omega_k \Rightarrow z_{i,t}^{s_1} = z_{i,t}^{s_2} = c_k^i, i = 1,\dots,I,$$

where c_k^i is the common decision for all units i up to time t for all the scenarios that are in the bundle Ω_k.

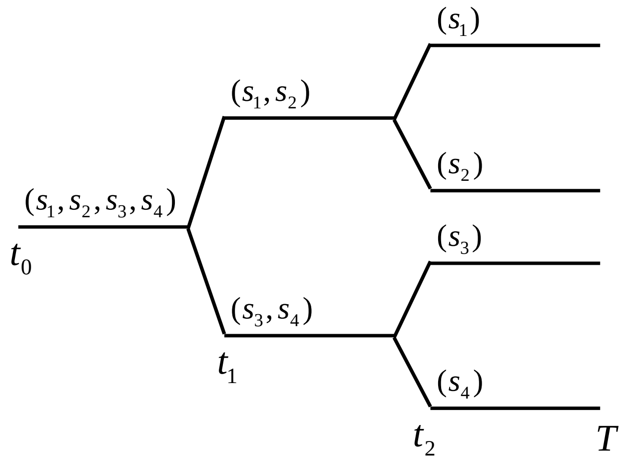

Figure 1: Scenarios tree

In 1998, Carøe and Schultz [2] proposed a two stage stochastic program for the unit commitment problem in a Hydro-thermal Power System. In this model, they divided the decision in long-term policies that have to be taken before the observation of load values and short-term corrections. Particularly, Start-up decisions for coal fired units will be considered in the first-stage, and start-up decisions for gas units and output levels for all units will be considered in the second stage. Then, given a set of I coal units and a set of K gas units, the formulation of the two stage program is,

$$\begin{aligned}
\text{Minimize: } & \sum_{i=1}^{I}\sum_{t=1}^{T} z_{it}F_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} y_{it}S_{it} + \\
& \sum_{s\in S} P_s \left[\sum_{i=1}^{I}\sum_{t=1}^{T} g^s_{it}C_{it} + \sum_{k=1}^{K}\sum_{t=1}^{T} g^s_{kt}C_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T} z^s_{kt}F_{kt} + \sum_{k=1}^{K}\sum_{t=1}^{T} y^s_{kt}S_{kt} \right]
\end{aligned}$$

$$\begin{aligned}
\text{Subject to: } & \sum_{i=1}^{I} g^s_{it} \geq d^s_t, && t = 1,\ldots,T, s \in S \\
& q_i z_{it} \leq g^s_{it} \leq Q_i z_{it} && t = 1,\ldots,T,\ i = 1,\ldots,I,\ s \in S \\
& q_k z^s_{kt} \leq g^s_{kt} \leq Q_k z^s_{kt} && t = 1,\ldots,T,\ k = 1,\ldots,K,\ s \in S \\
& y_{it} \geq z_{it} - z_{i(t-1)}, && t = 2,\ldots,T,\ i = 1,\ldots,I \\
& y^s_{kt} \geq z^s_{kt} - z^s_{k(t-1)}, && t = 2,\ldots,T,\ k = 1,\ldots,K,\ s \in S \\
& z_{it} - z_{i(t-1)} \leq z_{i\tau}, && \tau = t,\ldots,\min\{t + L_i - 1, T\},\ t = 2,\ldots,\ T, i = 1,\ldots,I \\
& z^s_{kt} - z^s_{k(t-1)} \leq z^s_{k\tau}, && \tau = t,\ldots,\min\{t + L_k - 1, T\},\ t = 2,\ldots,T,\ i = 1,\ldots,K,\ s \in S \\
& z_{i(t-1)} - z_{it} \leq 1 - z_{i\tau}, && \tau = t,\ldots,\min\{t + l_i - 1, T\},\ t = 2,\ldots,T,\ i = 1,\ldots,I \\
& z^s_{k(t-1)} - z^s_{kt} \leq 1 - z^s_{k\tau}, && \tau = t,\ldots,\min\{t + l_k - 1, T\},\ t = 2,\ldots,T,\ k = 1,\ldots,K,\ s \in S \\
& z_{it}, y_{it} \in \{0,1\}, && t = 1,\ldots,T,\ i = 1,\ldots,I \\
& z^s_{it}, y^s_{it} \in \{0,1\}, && t = 1,\ldots,T,\ k = 1,\ldots,K,\ s \in S \\
& g^s_{it} \geq 0, && t = 1,\ldots,T, i = 1,\ldots,I,\ s \in S \\
& g^s_{kt} \geq 0, && t = 1,\ldots,T, k = 1,\ldots,K,\ s \in S
\end{aligned}$$

We use these two models for solving some instances of the unit commitment problem, and we found that for the given instances the differences in the solution were not significant. In fact, the observed differences were less than 0.5% in all the cases, which are very small given that the maximum gap for CPLEX was set to 0.5%. The results of these experiments can be observed in Table 2. In this table the column Difference is the diference between the two objective functions, and the percentage is taken respect to the two stage formulation.

	Two Stages		Multiple Stages			
Scenarios	Time	Value	Time	Value	Diference	Percentage
1	6	1012949.88	6	1012949.88	0.00	0.00%
2	29	1031363.81	22	1030979.27	384.54	0.04%
3	38	1048659.60	27	1049123.33	463.73	0.04%
4	105	1038948.92	79	1040260.75	1311.83	0.13%
5	222	1042580.88	106	1042042.96	537.92	0.05%
6	321	1033536.29	170	1034359.06	822.77	0.08%
7	539	1032744.43	206	1033535.94	791.50	0.08%
8	689	1033529.30	1905	1034730.03	1200.73	0.12%
9	5804	1036795.47	561	1038309.29	1513.82	0.15%
10	4654	1041994.63	2037	1042599.71	605.08	0.06%

Table 2: Stochastic models comparison

4 Solution methods

For solving the stochastic unit commitment problem we tested three different methods. First, we use a Lagrangian relaxation where each bundle constraint has a multiplier λ_{it}^s associated with it. Then, a penalty term $\lambda_{it}^s(z_{it}^s - z_{it}^{s'})$ is added to the objective function, where s and s' are in the same bundle. When the bundles constraints are relaxed, the problem is separable in S different problems, which can be solved as independent deterministic unit commitment problems for each scenario. Therefore, one can use any strategy for solving the deterministic unit commitment problem for each scenario. In this experiment we used CPLEX 11.0 for solving each of the S problems. An interesting observation from this experiment is that each time that the bundles constraints are relaxed and the problem is solved the initial bound is already very tight. Then, after the Lagrangian relaxation converge the improvement in the bounds are not significant respect to the initial value. In Figure 2 the Lagrangian relaxation result for an instance of 32 units, 72 periods and 17 scenarios is presented. Additionally, in Table 3 results for some experiments are shown, where the last column shows the percentage improvement in the lower bound, after 100 iterations of the Lagrangian relaxation, for each instance.

scenarios	Bundle Relaxation	Best Bound	improvement
3	1047717.246	1047776.087	0.006%
4	1038184.112	1038211.470	0.003%
5	1041188.958	1041208.304	0.002%
6	1033397.135	1033422.672	0.002%
13	1036723.826	1036740.488	0.002%
16	1033032.104	1033055.840	0.002%
19	1032592.350	1032609.374	0.002%
22	1038030.051	1038048.192	0.002%
25	1042642.125	1042663.585	0.002%
28	1050592.102	1050602.840	0.001%

Table 3: Improvement in the lower bounds with the Lagrangian Relaxation

Second, we tried the heuristic method defined by Takriti and Birge in [8]. This is a similar technique to the Lagrangian, but the bundle constraints are written in the alternative form $z_{it}^s = c_{B(s,t)}$, where $c_{B(s,t)}$ is the value that need to be assigned to the ith decision variables, z_{it}^s, of all scenarios in the bundle $B(s,t)$ at

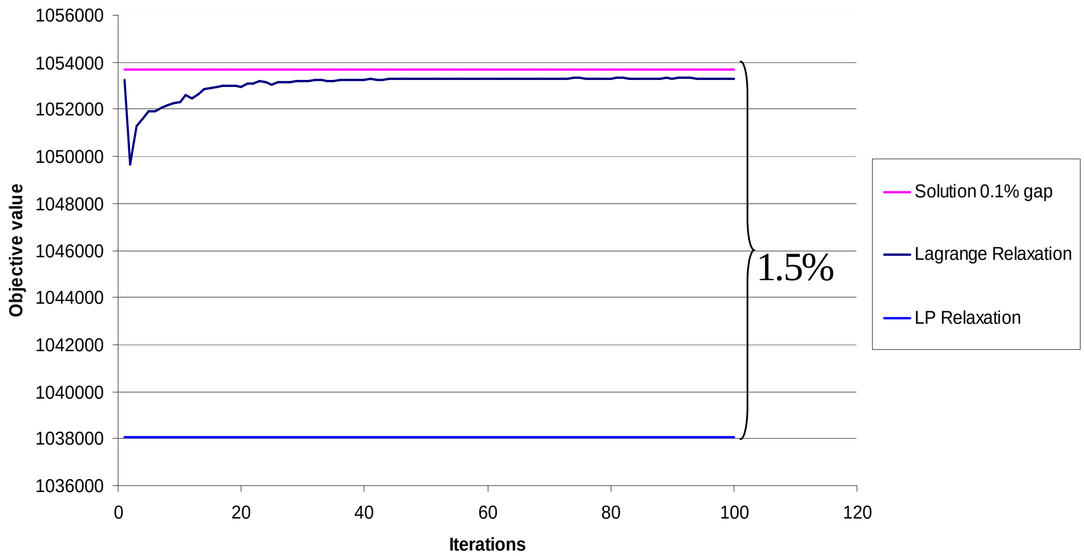

Figure 2: Lagrangian relaxation example

time t. The value of $c_{B(s,t)}$ is unknown, then in each iteration its value is updated as

$$c_{B(s,t)} = \frac{\sum_s P_s z_{it}^s}{\sum_s P_s},$$

and the convergence is achieved when the value of $c_{B(s,t)}$ approach to 1 or 0. In the implementation we use the suggested computation

$$\sum_{s=1}^{S} P_s \sum_{i=1}^{I} \sum_{t=1}^{T} |z_{it}^s - c_{B(s,t)}|$$

for defining the termination criteria. Then, once this quantity reach a predefined threshold the algorithm stops. An interesting observation is that this algorithm presented some cycling problems in some of the instance tested, see for example Figure 3. In that case is better to use the Lagrangian relaxation, as suggested by Takriti and Birge. Finally, once the algorithm reach the threshold, if the current solution is not feasible, one can continue the optimization process using branch an bound, or recover a feasible solution using some heuristic method, examples of such heuristics are given in [8]. In Figure 4 the progressive hedging results for an instance of 32 units, 72 periods and 6 scenarios is presented. In this instance the algorithm found the optimal solution after 81 iterations.

5 The LPRolling heuristic

In this section we will briefly explain the LPRolling heuristic. First, we observed that the linear relaxation for the problem was solved by CPLEX 11.0 relatively fast in all the cases we tried. For some of the instances, even with the 28 scenarios we had, CPLEX solved the linear relaxation in less than 1 second. Then, taken advantage of this characteristic we defined the LPRolling heuristic for finding a feasible solution for the stochastic problem. We solved the problem progressively, using the bundles structure of the scenarios tree. Then, given the list of bundles in the scenarios tree, we defined the first bundle as that one which its starting time was the time 1. For the first bundle we keep the binary constraints for the up/down variables in that bundle an relax the binary constraints everywhere else in the model. Once that problem is solved, the value of the up/down variables in the first bundle are fixed for all the scenarios, and the procedure

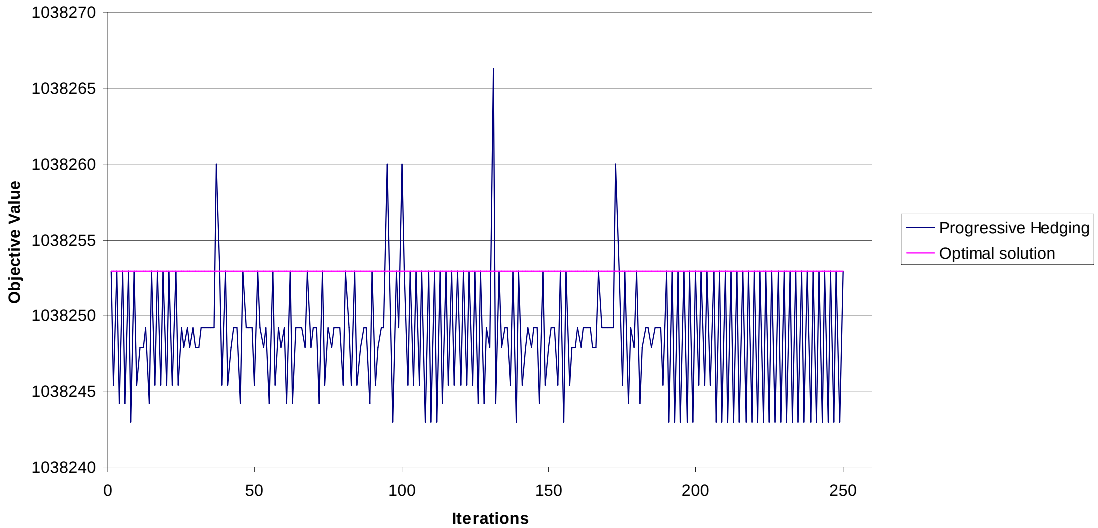

Figure 3: Progressive hedging cycling example

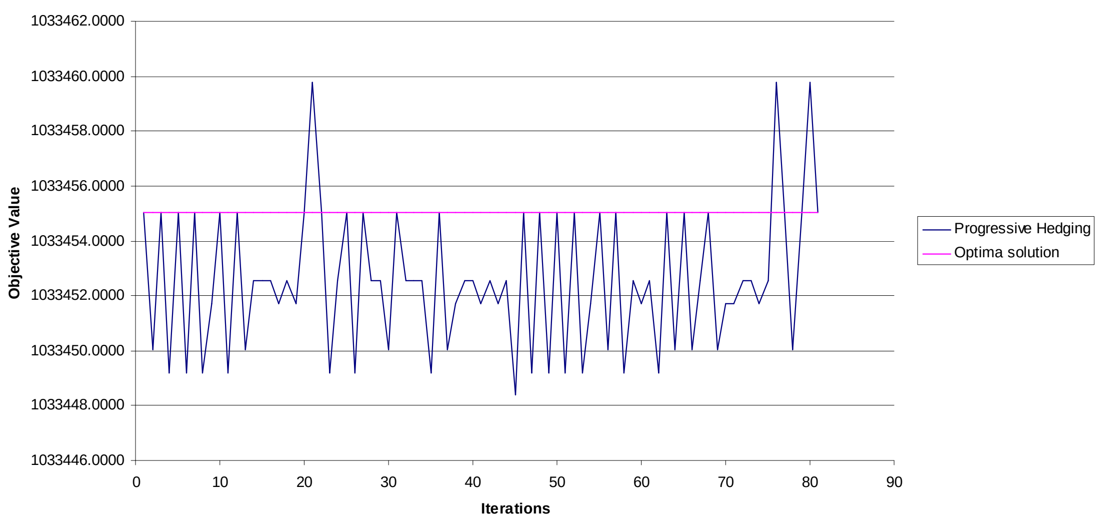

Figure 4: Progressive hedging convergence example

is repeated for the next bundles. That is, the binary constraints are keep it for the up/down variables in the current bundle. Then, the values of the up/down variables are fixed in all the scenarios for the time periods before the starting time of the current bundle. Finally, the binary constraints are relax for the bundles beyond the current bundle, and the new problem is solved. This process is repeated until all the bundle have been visited. This simple procedure has shown good results for the instances that have been solved during our experiments. Then, in Table 4 we have the results for an instance with 32 units and 72 periods. In this table we compare the results given by CPLEX 11.0 with a gap of 0.1% and the results of the LPRolling algorithm. These results show a significant improvement in the solutions times as well as an improvement in the objective functions values, as can be seen in the last column of the table, where we have the improvement percentage. Additionally, we ran two different variations of the instance. First, in results of Table 5 we introduce a random perturbation for the demand scenarios in order to increase the variability among the scenarios. Second, in results of Table 6 we modified the linear cost C_i across the units to decrease the big differences between the coal units and the gas units. In these tables the last column shows the optimality gap computed using the bound found from the bundles relaxation. Particularly, what we found in the last instance is that the modified cost increase significantly the solutions times, making harder for CPLEX to find the optimal solution. Additionally, the solutions found using the stochastic formulation for a gap of 0.1% are better than the solutions found by using the LPRolling heuristic, which explains the negative values in the column difference. However, looking at the computed gaps, the LPRolling results looks still competitive.

Scenarios	LPRolling	Time	Gap 0.1%	Time	Difference
3	1128645.430	17	1128647.090	41	0.000%
10	1080440.390	96	1081042.998	932	0.056%
17	1053400.010	235	1053688.109	682	0.027%
24	1061642.340	338	1061842.060	1113	0.019%

Table 4: LPRolling results

scenarios	Gap 0.1%	Time	LPRolling	Time	Difference	Bundle Relax	Gap
3	1128647.090	41	1128645.430	17	0.000%	1128585.575	0.005%
10	1081042.998	932	1080440.390	96	0.056%	1080392.637	0.004%
17	1053688.109	682	1053400.010	235	0.027%	1053229.820	0.016%

Table 5: Results with perturbed demand

scenarios	Gap 0.1%	Time	LPRolling	Time	Difference	Bundle Relax	Gap
3	696916.168	14807	697212.220	373	-0.042%	696846.670	0.052%
10	693550.887	3659 (0.33*%)	694349.870	716	-0.115%	693437.743	0.132%
17	689904.618	3641 (0.39*%)	690269.460	1778	-0.053%	689441.121	0.120%

* This is the best gap found after a limit time of one hour.

Table 6: Results with modified costs

6 Conclusions

In this section we present some conclusion about the review of the stochastic unit commitment problem and some solution methods for solving it, and the LPRolling heuristic. First, this review showed that the use of a tight formulation, as that presented by Rajan and Takriti in [6], can improved significantly the solutions times. Second, the experiments executed here did not show a significant differences on the solutions found using the stochastic two stage, and the stochastic multi stage formulations. Then, a two stage formulation for the problem could be enough for solving it. Third, using the Lagrangian relaxation we found that the relaxation of the bundles constraint gives a tight lower bound for the problem. Then, the improvements in the lower bound found with the Lagrangian relaxation were not significant. Fourth, the progressive hedging heuristic showed cycling problems during the experimentation. Fifth, the LPRolling heuristic showed improvements in solution times with competitive optimality gaps. However, we observe that the perturbations introduced on the demand scenarios and cost coefficients can significantly affect the solution times. Finally, and in important characteristic of all these methods is that they can be implemented in parallel, which can help to improve the solutions times for all of them.

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